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RELATIVITY TRANSFORMATION OF AN OSCILLATION INTO A TRAVELING WAVE, AND DE BROGLIE'S POSTULATE IN TERMS OF VELOCITY ANGLE

By Vladimir Karapetoff.

ABSTRACT

The author's previously given method of representation of space-time relationships in restricted relativity, by means of oblique coordinates, is briefly reviewed (J.O. S. A. and R. S. I., *13*, p. 155; 1926). The theory is then applied to the following problem: Two observers, S and S′, are moving at a relative velocity q. The S′ observer has in his system a set of pointers distributed parallel to q, and oscillating in synchronism and in phase with one another. It is required to describe the motion as seen by the observer S. By drawing two sine waves in the Lorentzian plane, it is shown graphically that for the S observer the pointers do not swing in synchronism, but form a traveling wave whose velocity of propagation, u, is connected with q by the familiar relationship $uq=c^2$, where c is the velocity of light.

Introducing the so-called velocity angle, α, determined by the relationship $\sin\alpha=q/c$, various other properties of the traveling wave, as seen by S, are deduced, such as the apparent frequency at a point in space, the apparent amplitude and its velocity of propagation, the Fitzgerald contraction of the oscillating system, and the slowing down of the frequency of oscillation.

By assuming a large number of S′ systems whose velocity angles differ from each other by very small amounts, the group velocity of the "super-light" waves is shown to be equal to the velocity q of the original S′ system. Finally, de Broglie's quantum principle is stated and transformed in terms of the velocity angle α.

Some time ago the writer showed that the time-space relationships corresponding to Einstein's restricted theory of relativity (with rectilinear motion) may be represented graphically by means of two systems of oblique coordinates.[1] It will now be shown that the relativity transformation of a stationary harmonic oscillation into a traveling

[1] V. Karapetoff, Straight-line Relativity in Oblique Coordinates; also Illustrated by a Mechanical Model; J. O. S. A. & R. S. I., *13*, p. 155; 1926.

wave may also be conveniently visualized by means of a similar diagram. This transformation has of late acquired a particular significance because of de Broglie's wave mechanics, and for this reason its graphical interpretation may be of interest to theoretical physicists.[2] The fundamental method of graphical representation of the Lorentz-Einstein transformation in oblique coordinates is shown in Figs. 1 and

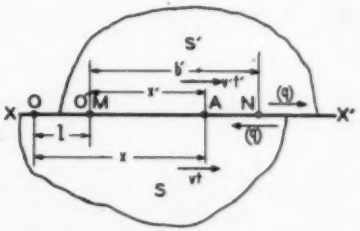

FIG. 1. *Relative motion of systems S and S' along XX'.*

2. There are two observers, S and S′, whose relative velocity is q. The S observer uses a system of oblique coordinate axes, OT and OX, to record time and to plot distances in the direction q. The S' observer uses the oblique axes OT' and OX' to record space-time measurements in his system. The angle XOT is greater than 90° by an amount α such that

$$\sin \alpha = q/c \tag{1a}$$

where c is the velocity of light in vacuo.

In the writer's article referred to above, as well as in this article, the velocity of light is taken as a unit, so that $c=1$. For example, if one second is the unit of time, the unit of length is equal to 300,000 km. Moreover, when using the oblique axes shown in Fig. 2, time and space are plotted to the same linear scale so that if, for example, in a drawing 5 cm parallel to the T axis represents one second, 5 cm along the X axis must represent 300,000 km.

The assumption $c=1$ simplifies eq. (1a) to

$$\sin \alpha = q \tag{1}$$

where q now stands for a numeric which expresses the relative velocity of the two observers as a fraction of the velocity of light.

Similarly, the angle $X'OT'$ is less than 90° by the same amount α. Incidentally, it will be seen that the directions OX' and OT are perpen-

[2] Louis de Broglie, La Mécanique Ondulatoire, p. 2; Paris, 1928; Arthur Haas, Materiewellen und Quantenmechanik, p. 18, Leipsic, 1929.

dicular to each other, and so are the directions OX and OT'. This fact simplifies the drawing of ordinates in the two systems.

The combination of the two systems of oblique coordinates shown in Fig. 2 has been named by the writer the Lorentzian Plane, and will be

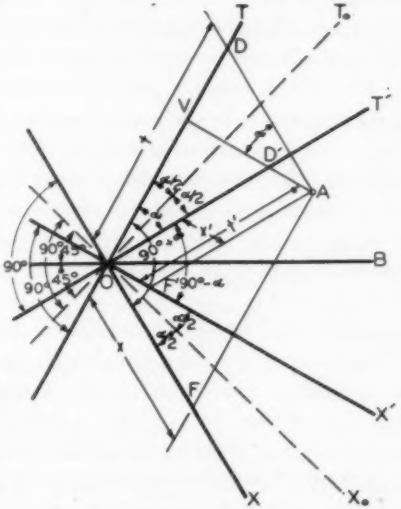

FIG. 2. *An event represented by point A referred to two sets of oblique axes forming a Lorentzian plane.*

later referred to as such. From the definition of $\sin\alpha$ as equal to q/c, it follows that

$$\cos\alpha=\sqrt{1-(q/c)^2} \tag{2}$$

In other words, the angle α has been introduced to replace by its cosine the familiar radical in the theory of relativity, which radical gives the amount by which lengths in the "other" system are shortened (the Fitzgerald contraction) and clocks are slow.

Let the observer S record a distance, $OF=x$ (Fig. 2) and an instant of time, $OD=t$, for an instantaneous event, say a flash of light, and let him plot x and t in his system of coordinates, obtaining the point A. Let the observer S' measure his x' and t' *for the same event* and plot the measured values $OF'=x'$ and $OD'=t'$ along his oblique axes. The point A so obtained will be the same as for the first observer. It is assumed, of course, that the world in which the two observers are experimenting is governed by the laws of Einstein's restricted theory of

relativity. In other words, the graphical construction just described is equivalent to the system of Lorentz-Einstein analytical equations of space-time transformation, as has been proved in the article already referred to.

Thus, the two systems of oblique coordinates in Fig. 2 satisfy the condition that an event correctly represented in one of the systems by one or more points is also correctly represented in the other system by the same points. Usually an event is given in terms of one of the observers. Having represented it in the corresponding system of oblique coordinates in Fig. 2, one can immediately "see" the same event from the point of view of the other observer, by drawing coordinates parallel to his system of axes. Numerous examples of events and transformations will be found in the preceding article, where it is also shown that all the results so obtained agree with the well known statements in the elementary theory of relativity.

Using the Lorentzian plane, we shall now analyze the following event. The observer S' places in his system a horizontal shaft parallel to the direction of relative motion XX' (Fig. 1), and at different points of the shaft he mounts eccentrics on it. The eccentricity vectors may be of different magnitude, but must all be parallel to each other. Each eccentric is connected by a suitable mechanism to a pointer which is thus made to perform a harmonic motion when the shaft is rotating.

Thus, the observer S' realizes in his system a number of material points which perform harmonic motions at one and the same frequency, say f' cycles per second, and which are in time phase with one another. The amplitudes of the individual motions may be different. Because all the motions are in phase with one another, there is no traveling wave, at least from the point of view of the observer S', but only pure oscillations. To put this statement in other words, the velocity of phase propagation is infinitely great, the phase of the motion at a given instant being the same for all the pointers. The problem is to describe the motions of the pointers as seen by the observer S.

According to the general method given above, we simply have to represent the motions of the pointers in the Lorentzian Plane, in the system of coordinates $X'OT'$ (Fig. 3), that is, from the point of view of the observer S'. The same graph will then give the motions as seen by the observer S, provided that we measure time and distances parallel to the axes OT and OX respectively. We shall choose the pointer M located at the origin O' (Fig. 1), and another N, at a distance b' from it. In Fig. 3, the harmonic motion of M is represented by the sine

curve *AJCDE*, time being plotted as abscissas along OT', and deflections from the middle point as ordinates, parallel to OX'. The sine wave looks somewhat unusual because of having been plotted in oblique coordinates. The frequency of the motion being f', the length of the base of the curve, AE, or the time of one complete oscillation, is $T' = 1/f'$. The amplitude of the motion is h'.

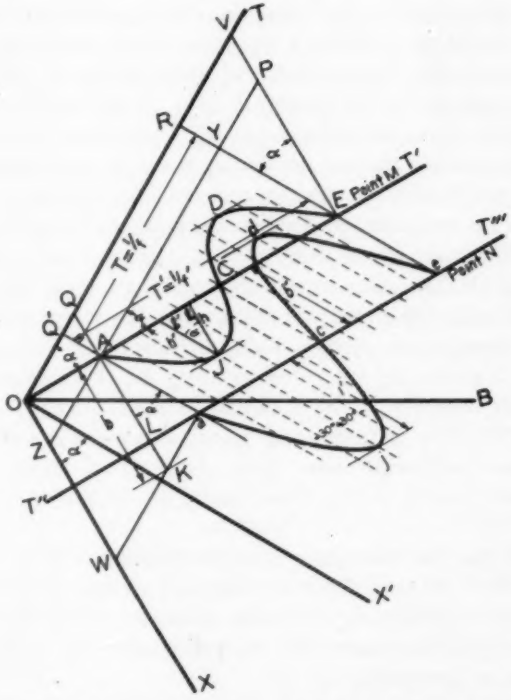

FIG. 3. *Transformation of a stationary oscillation into a traveling wave in the Lorentzian plane.*

The motion of the pointer N is represented by another sine wave, *ajcde*, of amplitude i', its base, $T''T'''$, being shifted with respect to OT' by the length b' parallel to OX'. The combination of the two sine waves represents the motion of the system MN, of length b' (Fig. 1), as seen and measured by the observer S'.

According to our fundamental assumption, the same two sine waves represent correctly the motion of the points M and N as seen by the

observer S, only we must measure distances parallel to OX and time parallel to OT. By so doing we find the following results:

(1) *The motions of the pointers are out of phase with each other and constitute a traveling wave.* Consider the point A (Fig. 3) and the two straight lines, $Q'a$ and QK, passing through it. The line $Q'a$ is parallel to the axis OX'; therefore, for the S' observer the events represented by the points A and a take place at the same instant of time, characterized by the coordinate OQ'. The line QK is parallel to the axis OX; consequently, for the S observer, the events represented by the points A and a occur at the instants WK and Wa respectively. For him, the passage through zero of the pointer N (Fig. 1) lags behind that of the pointer M by a time interval equal to Ka. The same result is true for the instants at which the two oscillating pointers reach their maxima deflections, and in fact for any two corresponding points.

If there are a large number of oscillating pointers driven by the shaft, each one will appear to the observer S to lag more the farther it is from O' (Fig. 1). In other words, the combination of the oscillating pointers will behave like a true traveling wave. To determine *the velocity of its phase propagation*, we resolve the "spatium" $Aa=b'$ into its time component, Ka, and the length component, AK, both from the point of view of the observer S. As is explained in the author's article previously referred to, a spatium in the Lorentzian plane is a vector which is partly space and partly time. Thus, Aa is a pure length for the S' observer, but is partly a length and partly an interval of time for the S observer.

For the S observer, the event under consideration at a, that is the passing of the oscillating pointer N through its zero point, occurs at a distance AK from the origin O', and at a time interval Ka later than a similar event for the pointer M. Therefore, the velocity, u, of phase propagation, as determined by S, is

$$u=AK/Ka=1/\sin\alpha. \tag{3}$$

Comparing this result with eq. (1), we find that

$$u=1/q. \tag{4}$$

In other words, the velocity of phase propagation is the reciprocal of the velocity q of the moving system itself. If the velocity of light, c, were not put equal to 1, eq. (4) would have become

$$u=c^2/q. \tag{4a}$$

This agrees with the statements by de Broglie and by Haas in the books referred to above. From the point of view of the theory of relativity, the transformation of a group of oscillations into a traveling wave is a transformation of a velocity which is infinite for S' into a finite velocity, u, for S.

(2) *The apparent frequency of oscillation, f, at a given point in space S is higher than f' in the ratio of* 1 *to cos* α. Instead of two pointers, M and N, consider now a very large number of pointers, close to one another, and all driven at the same frequency from the same shaft. In Fig. 3, the line $Q'a$ expresses the fact as at an instant OA (from the point of view of S') all the pointers pass through their respective zero positions. Similarly, the parallel line Re expresses the fact that at the instant OE the pointers also pass through their zero positions. The time interval, AE, between these two lines (or loci) is the period of oscillation, T', as determined by the observer S'.

According to the general method agreed upon, these two parallel lines have the same meaning for the S observer, only he will measure the time interval between these lines parallel to his axis OT, thus obtaining the value $AY = T = 1/f$. It will be seen from the figure that

$$AY = AE \cos \alpha \tag{5}$$

or

$$f = f'/\cos \alpha = f'/\sqrt{1-(q/c)^2}. \tag{6}$$

This result agrees with the formulas given by de Broglie and Haas (loc. cit.). The observer S interprets the frequency f and the period T as referring to a stationary point in his space, so that the oscillation which he has in mind in reality is formed by numerous portions of oscillations of the actual pointers as they pass by this point in the S space.

(3) *The amplitude of oscillations is reduced and is propagated at the velocity q.* Let the observer S follow a particular pointer, say M, in its motion through space, by watching its zero position, O', and measuring the amplitudes, h, of its deflections on a scale calibrated in centimeters in the S' system. When the deflection is $h' = g'J$, the S observer reads the length

$$gJ = h = h' \cos \alpha. \tag{7}$$

This is the familiar Fitzgerald contraction of lengths when measured parallel to the direction of relative motion. Thus, the true intensity of oscillations is reduced for the S observer.

Since the point O' moves at a velocity equal to q with respect to the S observer, the amplitude h also seemingly progresses at this velocity. Consider the points A and E. For the S' observer, the spatium AE is a pure time interval equal to a period of oscillation. We may also think of the points A and E as of the two consecutive instants at which the pointer M passes through the origin O'.

For the S observer, AE consists of the length PE and the time interval AP. For him, the two coincidences of M and O' take place at a time interval equal to AP, and the point O' has moved between these coincidences by the amount EP. Assuming the motion of the origin O' to be uniform, its velocity is

$$PE/AP = \sin \alpha = q \tag{8}$$

as was to be expected from the theory of the Lorentzian plane.

It is of importance to differentiate between the velocity (q) of propagation of the amplitude of the traveling wave and the velocity ($u = 1/q$) of propagation of its phase. The two are the reciprocals of each other. This fact is somewhat difficult to visualize physically, and the diagram shown in Fig. 3 may help to see it at least geometrically. The velocity of the amplitude is determined from the triangle AEP, that of the phase from the triangle AKa.

It may help to obtain a physical conception of what is actually taking place by remembering two facts: (a) The amplitudes are attached to the moving material system and therefore move at the velocity q of that system. It is not the amplitude itself that moves, as if it were a geometric line, but its occurrence. (b) In an oscillation, such as S' sees it, the velocity of propagation of the phase is infinitely great, all portions of the system being at the same phase simultaneously. Relativity of motion makes the observer S see the more distant portions of the system S' later than its more proximate parts, and this reduces the velocity of phase propagation to a finite, although very high, value u.

The foregoing deduction shows that u is a reciprocal of q, so that as q increases from zero to c, the phase velocity, u, decreases from infinity to c.

(4) *The Fitzgerald contraction of the oscillating system.* It has been pointed out above that the line Aa of the S' observer, is a spatium for the S observer, with the length component $AK = b$. Since b is longer than b', this result may at first seem to contradict the familiar contraction of a distance when measured by the "other" observer. However,

from the S point of view, it is necessary to distinguish between two entirely different "events." One event is the passing of the two pointers, M and N, through their zero positions, and the problem for S is to find the distance between these zero positions at those particular instants. The answer is that the distance is AK and that the pointer N passes through its zero position later than M, the time interval between the two being Ka.

The other "event" is the distance between the zero divisions on the two scales, irrespective of the pointers, and the problem for S is to find this distance. The locus of the motion of the zero division of the scale M is OT', because this division coincides with the origin O' and hence for it $x'=O$. The locus of zero division of the scale N is the line $T''T'''$, at a distance b' from it.

To the S observer, these two lines are loci of the two points moving uniformly at the velocity q. To find the distance between the moving points, he measures the distance between the loci, parallel to OX. This distance is AL which is smaller than b' and is equal to b' Cos α. This is the correct expression for the Fitzgerald contraction.

Thus, a distance b' in the S' system becomes either $AK>b'$ or $AL<b'$, depending on the particular event which the S observer is following; for the S' observer it is always the same distance b'.

(5) *The slowing down of the frequency of oscillation.* It is well known from the elementary theory of relativity that the clocks in the "other" system seem to be slow. On the other hand, we have previously found that to the S observer the frequency f' becomes a higher frequency, f; eq. (6). Here again, we have to distinguish carefully between two events which S may follow. To see this difference, we shall again assume that the shaft operates a very large number of pointers placed very close to each other. As a limit, the number of pointers may become infinitely great, in which case there is a definite amplitude and frequency of oscillation at each point of space, moving with the observer S'.

One of the events which the S observer may be asked to follow is to determine the frequency of oscillation at a given point of his own space, that is, of the space stationary with respect to him, even though different portions of this oscillation are produced by different elementary pointers moving with S'. As explained above, the event in question is determined for the S' observer by the combination of the lines Re and $Q'a$, the time interval between which, measured in the S' system, is $T'=AE$. The S observer measures the time interval in the direction OT and obtains a smaller value, $AY=T$.

The other event which the observer may be asked to take measurements on is the time of oscillation of a particular pointer as it swings back and forth while moving with the S' system. For the S' observer this event is represented by the spatium AE or ae, both of which to him represent a time interval T'. The S observer resolves this spatium into its length component, EP, and its time component, AP. Thus, to him the time interval in question is

$$AP = T'/\cos \alpha. \tag{9}$$

This is the correct slowing down of a clock according to the theory of relativity. The S observer reports a lower frequency of oscillation of individual pointers than S', but a higher wave frequency at a fixed point in his own space.

GROUP VELOCITY

In place of the above described single S' system, with its oscillation apparatus, imagine now a large number, say $2N+1$, of such systems, identical in all respects, all moving at the same velocity q, and each provided with an oscillation device vibrating at an identical frequency f'. Let also all the devices vibrate in space phase coincidence with each other. The observer on the original S' system will see a number of systems identical with his own and stationary with respect to himself. The S observer will see the phenomenon described in Fig. 3 repeated $2N+1$ times in space.

Now let the original S' system continue to move at the same velocity q, and let N other systems be slowed down by infinitesimal amounts $dq, 2dq, \ldots, Ndq$. Let the remaining N systems be speeded up by similar amounts. Each system, after having reached the new velocity, is to continue to move uniformly at that velocity. Under the new established conditions, the S' observer will see N systems slowly creeping past him at different speeds in one direction, and the remaining N systems in the other direction. By assumption, oscillating pointers are spaced very close to each other in each system, so that as S' watches one of his pointers, he will see the zero division on the scale of a pointer of an adjacent system creep by and another take its place. Since the velocity of creeping is very small as compared to the average velocity of oscillation of the pointers, the space coincidence of phase among the systems persists for a long time.

On each of the $2N+1$ systems, the S observer sees the phenomenon shown in Fig. 3, only for the different systems the velocity angles between the axes are

$$\alpha,\ \alpha+d\alpha,\ \alpha+2d\alpha,\ \text{etc.},\ \alpha+Nd\alpha\ ;$$
$$\alpha-d\alpha,\ \alpha-2d\alpha,\ \text{etc.},\ \alpha-Nd\alpha.$$

He sees the systems creep past each other, he sees each wave moving at a slightly different velocity u, the space phase coincidence being only approximate. However, since the differences in angle α are very small, the sine waves shown in Fig. 3, when drawn for the different systems, nearly coincide. Moreover, since the velocities are grouped symmetrically about the original velocity q, the grouping of infinitesimal phase differences is symmetrical with respect to the original system S' and *moves with it*. This means that to the S observer the places at which there is a seeming coincidence in phase (for a certain period of time) appear moving at the velocity q, although the phases of the waves themselves move at velocities u, $u+du$, etc.

The velocity at which the region of phase coincidence of several waves appears to move is known as the group velocity. We thus reach the conclusion that the group velocity in the case under consideration is equal to the actual average velocity of the material system consisting of $2N+1$ systems of the type S'.

To show the same result analytically, we note that to the S' observer the time phase of his oscillations follows the law

$$\sin f't'. \tag{10a}$$

To represent the same oscillations from the point of view of the S observer, t' must be expressed in terms of t and x. In Fig. 2, $OV=OD-VD$, or

$$t'\cos\alpha=t-x\sin\alpha. \tag{10}$$

The coordinates in question are those of an arbitrary point A, so that any other point could have been chosen; the relationship (10) is quite general. Thus, for the S observer, the phase of oscillations follows the law

$$\begin{aligned}\sin f't' &= \sin\left[(f'/\cos\alpha)(t-x\sin\alpha)\right]\\ &= \sin\left[f'(t\sec\alpha-x\tan\alpha)\right].\end{aligned} \tag{11}$$

The distances x and the instants t at which the oscillations belonging to the different S' systems are in phase with one another satisfy the condition that the expression in the brackets in eq. (11) remains constant when α is varied. This is because we wish to pick out the oscillations which are in phase with one another, and for such oscillations the

sine of the angle (and consequently the angle itself) must be constant. Since f' is a constant, we have:

$$t \sec \alpha - x \tan \alpha = \text{const.} \tag{12}$$

Differentiating with respect to α and cancelling the common factor, gives

$$t \sin \alpha - x = 0 \tag{13}$$

or

$$x = t \sin \alpha = tq.$$

In other words, the point x at which the oscillations remain cumulative moves at the velocity q equal to the velocity of the initial or "average" system S'. This is an analytical proof of the fact that the group velocity of waves is q.

DE BROGLIE'S QUANTUM PRINCIPLE

The two fundamental assumptions of de Broglie's wave mechanics are as follows:

(1) A material particle of mass m and velocity q is in some manner associated with a system of waves traveling at a velocity $u = c^2/q$, greater than that of light. The group velocity of these waves is equal to the velocity q of the particle itself.

(2) The wave length, λ, of these waves is determined by the condition that the "action" of the particle, per wave length, is equal to Planck's action constant, h.

The second postulate may be written in the form

$$mq\lambda = h. \tag{15}$$

It is usually expressed in the form

$$mc^2 = hf \tag{16}$$

where f is the frequency of the de Broglie waves for a stationary observer. The two forms are equivalent, as may be seen from the following considerations: Multiplying both sides of eq. (15) by f, we obtain

$$mq\lambda f = hf. \tag{17}$$

But λf is equal to the velocity u of the waves. According to eq. (4a), $qu = c^2$, and eq. (16) follows directly.

It is of interest to express de Broglie's assumption (15) in terms of the velocity angle, α, so as to connect his theory with the diagram shown

in Fig. 3. If the mass of the particle at rest is m_0, its mass at the velocity q is

$$m = m_0/\sqrt{1-(q/c)^2}. \tag{18}$$

Substituting for q/c its value from eq. (1a), gives

$$m = m_0 \sec \alpha. \tag{18a}$$

The expression (15) therefore becomes

$$\lambda m_0 c \tan \alpha = h. \tag{19}$$

Thus, the wave length corresponding to a velocity angle α and a rest mass m_0, is

$$\lambda = N^{-1} = (h/m_0 c) \cot \alpha \tag{20}$$

where N is the number of waves per unit length.

The frequency of the waves may be determined from eq. (16), eliminating m by means of eq. (18a). The result is

$$f = (m_0 c^2/h) \sec \alpha. \tag{21}$$

It is hoped that eqs. (20) and (21) may prove useful in the further development of wave mechanics on the principle of relativity.

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"Spectra," by R. C. Johnson, with a general preface by O. W. Richardson, E. P. Dutton and Company, Inc., New York, $1.15.

"Spectra" is one of a series of "Monographs on Physical Subjects," published by E. P. Dutton & Company. That the subject of spectra should be treated in 100 small pages demands that brevity be the keynote in the discussion. And so it is in "Spectra." Under the headings of "Line Spectra" and "Band Spectra," the two main chapters contain a rather sketchy semi-theoretical review of the chief problems involved in the study of spectra. The interpretation is, unfortunately, completely from the point of view of atomic theory previous to the rise of the quantum mechanics, although the book was written in 1928.

An elementary treatment, of the same standard as the preceding descriptive and theoretical discussion, of the experimental technique in spectroscopic investigation closes the monograph.

M. MUSKAT

La Diffusion Molèculaire de la Lumiére Dans les Fluides. Jean Cabannes; viii+326 pp. Les Presses Universitaires de France, Paris. 65 fr.

This volume represents an attempt to bring together in one place all that which may still be considered pertinent to the physics of the molecular scattering of light. The experimental and theoretical researches of the authors are included and they serve well in bringing the subject up to date, except in so far as quantum theory considerations might be applicable to some of the phases of it.

The scattering of light by a single molecule is first derived, and this is applied to a perfect gas with the assumption that the molecules scatter incoherently. Taking the ideal transparent medium as being composed of a system of anisotropically polarizable molecules, the theory of the propagation of light in such media is presented. Besides explaining in this way the dispersion and scattering of light, expressions are derived for the electric and magnetic birefringence and magnetic rotatory polarization.

Chapters III and IV give experimental methods and sample data for experiments testing the previously developed theory, as well as for the measurement of the depolarization of the scattered light.

The problem of molecular optical anisotropy is then discussed theoretically and experimentally. Applications of the problem to the matter of molecular symmetry properties and structures are given.

The general topic of light diffusion in and scattering by liquids is treated next. A critique of both the molecular theories, in which fluctuation methods are applied, and the thermodynamic theories is given. Interesting data and discussion is then presented on the change of molecular anisotropy with change of state from gaseous to the liquid form.

In the next to the last two chapters Rocard presents a theoretical study and critique of previous theories of the phenomena of critical opalescence, after developing a more exact evaluation than has previously been obtained of the internal fields within a fluid. The last chapter suggests a series of experiments to test Rocard's theory and for further research in the general field.

A well organized and complete bibliography follows the text.

For those at all interested either in the experimental or theoretical phases of this topic, the book should prove useful as a guide and stimulus for further research and investigation.

M. MUSKAT

INSTRUMENT SECTION

INSTRUMENTS AND METHODS USED FOR MEASURING SPECTRAL LIGHT INTENSITIES BY PHOTOGRAPHY*

By George R. Harrison

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I. INTRODUCTION

With the growing importance of precise measures of light intensity, particularly in spectroscopy, increased attention is being paid to the development of apparatus and procedure designed to make determinations of intensity accurate, convenient, and rapid. Three objective methods, which may be designated respectively the radiometric, the photoelectric, and the photographic, are available. Each one of these has certain advantages, and it is the purpose of the present article to discuss the type of investigation in which the photographic method is most useful, comparing critically the various types of photographic spectrophotometry which have been proposed.

For the direct comparison of spectral lines of sufficient intensity from constant sources the thermopile or thermocouple is most convenient, for its response is definite, and is proportional to the incident light intensity at all wave lengths. It can be made as sensitive as the Nichols radiometer, the bolometer, or the radiomicrometer,[1] which are the most important of the other sensitive radiation meters which give a response independent of the wave length of the light being measured, and it is the most easily handled. Some one of these is always the court of last resort in measuring spectral light intensities, for when in heterochromatic photometry a photographic or a photoelectric instrument is used, it must be standardized by reference to some non-selective device. The thermo-relay of Moll and Burger[2] has greatly increased the use-

fulness of the vacuum thermo-element. Radiometers, however, are generally usable only with spectrometers of low resolving power. Their direct use in spectrophotometry is indicated when simple spectra from constant sources of great intensity are to be measured.

The photoelectric cell has been greatly improved in recent years, and is convenient for many problems involving measures of intensity. Cells can be obtained which are sensitive to any portion of the spectrum between 2000 A and 10,000 A, although no single cell is competent for that entire range. The dependence of the response on wave length remains the greatest drawback to the use of the photoelectric cell for spectrophotometry. The high sensitivity which is attainable, especially when use is made of vacuum tube amplifiers, and the practically instantaneous response, make the cell very useful for homochromatic photometry employed in various problems; as for instance, the determination of transmission or reflection factors of materials at various wave lengths.[3]

By far the most generally applicable method in spectrophotometry is that in which the photographic emulsion is the sensitive medium, its response being calibrated by means of the photoelectric cell or thermocouple, and standardized in terms of the thermocouple or other nonselective radiometer. By *calibration* I mean the determination of the dependence of response upon intensity at individual wave lengths in the spectrum; *standardization* refers to the determination of the variation of this dependence with wave length. In addition the degree of response of the emulsion must be measured by means of a densitometer or microphotometer which generally contains a thermocouple or photoelectric cell as the sensitive element. The chief advantages of the photographic method are, (1) the emulsion acts as a recording medium, giving a permanent fixed response to a large number of simultaneously incident intensities, (2) it acts as an integrating medium, summing up the total energy falling on it at a particular spot, and (3) it furnishes the sensitivity not available in the photoelectric cell or the thermopile.

Whenever high resolving power, or low light intensity, or a variable light source, or dozens of simultaneous readings are involved, the investigator is generally wise to use the photographic method. It still labors somewhat under a cloud because of the large number of photographic investigations performed in the past without adequate control. Nevertheless, the modern emulsion, if properly handled, is an extremely trustworthy and useful scientific tool.

I shall discuss the general principles of photographic photometry first, and afterwards the various approximations and simplifications applicable to special problems. Four references to general discussions of the subject may be given at this point. Ornstein, director of the physical laboratory at Utrecht, where many of the chief developments and applications of photographic photometry have been made in the past few years, has summarized in two papers the methods used and the results obtained in his institute.[4] Dorgelo has written a similar summary.[5] Dobson, Griffiths, and Harrison, in England, have published a book on the subject,[6] but its chief emphasis is laid on the wedge methods.*

II. THE GENERAL PRINCIPLES OF PHOTOGRAPHIC PHOTOMETRY

For many years it was believed that the photographic emulsion could be used only as a qualitative measuring instrument, for in practice it was found that apparently identical emulsions affected by identical light intensities would yield responses varying many percent. This undoubtedly arose from the large number of variable factors involved. It is now known that the best modern emulsions when properly handled give results reproducible to less than 3%, and as a large number of measurements of the same quantity can quickly be made, final values with a probable error smaller than 1% can be obtained by the photographic method.

When a photographic emulsion has been exposed to an incident beam of light, developed, washed, fixed, and dried, the consequent blackening is influenced by a multitude of factors. The chief of these are the intensity of the light I, the time of exposure t, the wave length of the light λ, the character of the emulsion, the kind of developer used, and the time, temperature, and character of development. Several minor causes of variation, including the temperature of the plate at the time of exposure, the rate of drying, and the geometrical position of the image on the plate, can be neglected for the time being.

The degree of blackening of the plate is generally measured by passing a beam of light through it, and determining the reduction in intensity of this beam. If we represent the incident intensity by i, (to distinguish it from the intensity I which originally produced the blackening), and the transmitted intensity by i', then i/i' is called the *opacity*, O, of the plate, and $\log_{10}O=d$, the density. It is usually more

* I am greatly indebted to Dr. K. K. Darrow for valuable criticism and advice in preparing the present article.

convenient to work with densities than with opacities, since they give simpler curves when plotted against the other variables.

In order to represent the curve connecting the different variables which control the emulsion response, at least a nine-dimensional figure would be required. Fortunately it is possible to keep a large number of these variables constant during any experiment. The density d and intensity I are fundamentally involved, as is the wave length λ.* We thus are chiefly interested in a curve in three-dimensional space connecting density, intensity, and wave length, if we can keep the time of exposure, the character of the emulsion, and the development constant.

In certain cases there is no need for direct comparison of light intensities associated with different wave lengths; where the variable λ does not enter we shall refer to the process as *homochromatic*. Where intensities in different spectral regions must be compared we shall speak of *heterochromatic* spectrophotometry.

What may be termed the fundamental principle of photographic photometry is that each plate or film must be individually calibrated, and standardized as well if the problem is one of heterochromatic photometry. In fact, for the most accurate work, several portions of every plate should be calibrated and standardized. Improvements in photographic methods have not enabled us to dispense with these precautions; rather they have afforded us more convenient ways of taking them.

Much work has been done on the form of the analytic function relating the different variables mentioned above.[7] This does not concern us here, since in practical work the numerous constants of the equation would have to be determined anew for each individual plate, and by using null methods we can achieve the desired result without knowing the equation. However certain qualitative relations between the variables are available to guide us in selecting the best working conditions, and these will now be discussed.

1. *Characteristics of Photographic Emulsions.* a. *The Characteristic Curve.* The photographic plate as ordinarily used commercially is usually exposed to white light for a small fraction of a minute. A large literature has grown up on the characteristics of emulsions under these conditions, but as yet little has been done on their behavior when exposed to monochromatic light.

* It is possible to use a constant density value, as is done in one type of wedge method described below; this however is only an apparent suppression of the variable.

To show the relation between the variables *light intensity* and *blackening of the emulsion*, when all other factors are kept constant, it is best to plot the density d against the logarithm of the intensity I, $\log_{10} I$. The curve is usually of the general type of those shown in Fig. 1. Often curves are plotted with the axis of abscissae marked $\log_{10} E$. The symbol E stands for the *exposure*, which is defined as the product It. In the experiments from which such curves are derived, I is generally kept constant and t is varied. Evidently such a curve is equivalent to one formed by plotting d against $\log t$, and is apt to be misleading for the purposes of exact photometry, since different pairs of values of I and t may produce different values of density even though the product It remains the same. This is described as the failure of the

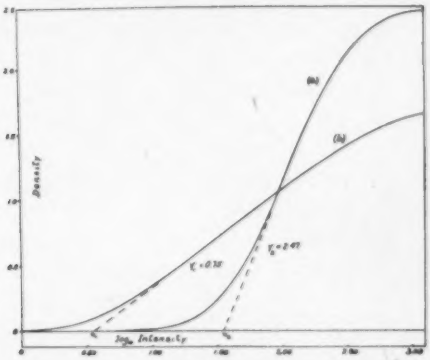

FIG. 1. *Characteristic curves obtained with monochromatic light on typical emulsions of* (*a*) *low speed and high contrast, and* (*b*) *high speed and low contrast.*

reciprocity law. Accordingly we will refer to curves of d against $\log t$ as *H & D curves*,[8] after Hurter and Driffield, who first plotted them, and to curves of d against $\log I$ as "*characteristic curves*." The use of $\log E$ as the independent variable is to be discouraged in spectral work except possibly in cases where the reciprocity law has been definitely verified over the whole range of conditions used. For spectrophotometric purposes characteristic curves should be determined for each wave length to be measured, by interpolation if necessary. In cases where light of mixed colors is used, the calibration curves must be made with light of the same quality as that being studied, to avoid errors due to the photographic Purkinje effect.[9]

The central portion of the type of curve shown in Fig. 1 is often a straight line which can be represented by the equation $d=\gamma \log I - n$

where γ, the slope of this line, is called the *contrast*, and n, its intercept on the log I axis when extended, is a measure of the *inertia*. Where both I and t are varied the relation of these to d can be represented by a *characteristic surface*, as shown in Fig. 2. The flat portion of this curve in three-dimensional space can be represented by the relation $d = \gamma \log I\ t^p - n$, which can also be written $d = \gamma \log I + p\gamma \log t - n$. The symbol p stands for what is known as Schwarzschild's constant;[10] in the case illustrated it has approximately the value 0.55. It is not safe, however, to assume p to be a constant for any particular emulsion, as has been shown by Abney and others.[11] In some emulsions p is

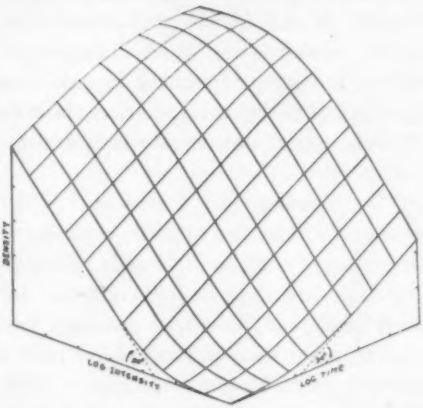

FIG. 2. *Graph in three dimensions showing a family of characteristic curves (d—log I) and a family of H. & D. curves (d—log t) for a typical emulsion. The contrast is 1.2, but from the d—log t curves the apparent contrast is only 0.66, making the value of Schwarzschild's p 0.55. This factor is not quite constant, however. Large errors would result from calibrating such an emulsion by varying the time of exposure instead of the intensity unless the values of p were accurately determined, and this is not usually worth while.*

nearly equal to 1 under conditions in which a practical density can be built up in a few seconds; for extreme values of t or I it is apt to differ considerably. For our purpose the following facts need emphasis: (1) It is not safe to consider p equal to 1 (i.e. to assume the reciprocity law), or to a constant (i.e. apply Schwarzschild's correction) without first determining it under the prevailing circumstances, a process which as a rule is not worth the trouble; and hence it is usually best to calibrate by varying I directly; (2) Under certain circumstances it is legitimate to calibrate plates with a time of exposure different from that which is to be applied in the measurements. With certain emulsion-developer

combinations the ratio of the two times of exposure may be made as great as 10 or 100, but the legitimacy of this must be ascertained by actual test. Where very long times of exposure are necessary in measuring low intensities, as in work on the Raman effect, one may wish to use factors as great as 1000 or more, but it is in precisely such cases that the failure of the reciprocity law is apt to be most serious.[12]

Returning to Fig. 1: The upper part of the curve, called the "shoulder," is known as the *over-exposure region*, and although the simple equation given above no longer applies, this region can still be used for photometry; similarly the "toe" or *under-exposure* region is useful. In both of these extreme regions, however, the accuracy of the results is somewhat reduced. It should be emphasized that whatever the form of the curve connecting some chosen function of the blackening with a function of the intensity producing this, one may use it to determine unknown intensities by interpolation, but the more closely it approaches a straight line of unit slope the more convenient and accurate will the process be.

b. *Variation of the Characteristic Curve with Type of Emulsion.* Emulsions can be roughly classified as *slow*, *medium*, and *fast*, according as the *inertia* is large, medium, or small. Similarly they are described has having high, medium, or low contrast, depending on the value of γ. *Contrast* and *speed* ($S=10/n$) generally vary in an inverse manner, so that a fast plate usually has low contrast and vice versa. High speed is generally given by plates with large grain size, which in turn decreases resolving power and increases the tendency to fog. This fog, which is a more or less even blackening of the plate on development even where no light has touched it, is a great drawback in photometry, as it decreases the sensitivity of measurement and introduces new errors which must be corrected for in ways which are as yet rather uncertain.

Certain types of plates are made with mixtures or successive coatings of two or more simple emulsions, which result in characteristic curves which may show an abrupt change of contrast near the middle of what would normally be the straight portion. This property is often advantageous for commercial photography for it gives increased speed with greater blackening for weak exposures, and greater contrast for strong exposures. Such emulsions can be used for photographic photometry, but they are undesirable because of the added uncertainty introduced in determining the exact course of the more complicated characteristic curve with a limited number of experimental points.

Speed ratings stated by manufacturers ordinarily apply to white light, but give a rough indication of the qualities to be expected in the plate when used for spectrum work. Fast plates are generally to be chosen when weak lines are to be photographed, or when weak and strong lines are to be made to appear with about the same density. The "latitude" of such a plate (the intensity range on the graph covered by the straight line portion) is long; this is an advantage, which, however, is generally nullified by the effect of the increased background and fog densities. A slow plate is to be chosen where high resolving power is needed, or where weak lines, background, and fog are to be eliminated, or where high contrast is desired for other reasons.

Plates of medium speed, such as the Eastman 36 or Seed 23, are the most suitable for general photometric purposes. They give the desired balance between latitude, contrast, lack of fog, and speed. It is sometimes stated that the greatest accuracy is obtained by having the contrast as high as possible, as this gives a large density range for a small intensity range, and makes an apparently great difference Δd between two close lines differing in intensity by ΔI. As in many other interpolation processes, however, the 45° curve of unit contrast can be shown to give the greatest accuracy when all the necessary steps are taken into account.

c. *Variation of Emulsion Characteristics with Wave Length.* The term *sensitivity* as applied to the photographic plate is very indefinite. An apparently sensible definition would be "the ratio of density produced to light intensity producing it" but this ratio varies greatly with either variable. Nevertheless it is desirable to adopt some measurable quantity to characterize the response of a plate to light of various wave lengths, and we may use this ratio for that purpose, imperfect as it is. It becomes somewhat more definite if we take as the measure of sensitivity the reciprocal of the intensity required to produce unit density in a given time. The type of curve obtained for an ordinary emulsion when this definition of sensitivity is made is displayed in Fig. 3. Had another density been chosen as standard this curve would appear somewhat different.

The spectral sensitivity characteristics of the ordiary emulsion may be described briefly as follows: In the blue green, the emulsion is very insensitive. The sensitivity rises rapidly as the wave length is diminished reaching a maximum in the ultraviolet near 3600A (and not in the visible near 4800A as one might expect from a casual interpretation of the data published by most plate manufacturers). Further on, it

diminishes slowly as far as the neighborhood of 3000A, where it often rises slightly again as far as 2500A, and then makes a rapid descent, falling to very low values at 2000 A. Various methods have been used to

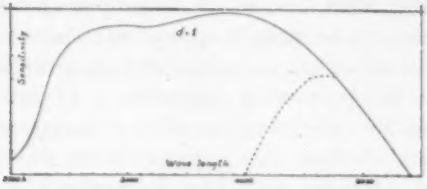

FIG. 3. *The solid curve shows the variation of sensitivity (defined here as the reciprocal of the light intensity required to produce unit density in a given time) with wave length for an ordinary emulsion under conditions such as are frequently met with in photometry. The dotted line shows roughly the course of the sensitivity curve obtained by inspection of the usual wedge photographs showing the sensitivities of photographic materials. The difference between the two is due to the fact that the latter curve has not been corrected for the absorption by the glasss in the neutral wedge and in the lamp used in making the photograph, nor for the variation of light intensity with wave length.*

change the general character of this curve, and especially to extend it beyond its usual limits. These include staining of the emulsion, decrease of the amount of gelatine in the emulsion, and coating of plates with fluorescent substances. The International Critical Tables, *5*, and the catalogs of manufacturers present wedge photographs showing roughly

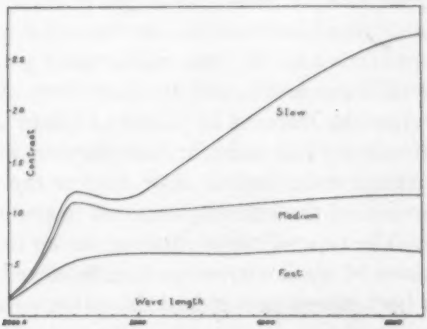

FIG. 4. *Curves showing the variation of contrast with wave length for three typical emulsions, respectively slow, medium, and fast.*

the sensitivity characteristics which the more important special emulsions display in the visible and infrared regions. These photographs are generally misleading below 4800A. Very little work on the sensi-

tivity of photographic materials has been done in the ultraviolet; some data are to be found in papers of Harrison[13] and of Jones and Sandvik.[14]

The sensitivity variation may be considered as the sum of two, one of which is to be described as the change of contrast, the other the change of speed. In Fig. 4 are shown curves giving the variation of contrast with wave length for several representative emulsions. Similar curves showing the variation of speed with wave length can be found in reference 13. It should be emphasized that the data given are of value for qualitative purposes only, and should never be used for accurate photometric work. No royal road has yet been found by which the experimenter can detour around the necessity for determining the corrections which must be applied to his results in spectrophotometry on account of the sensitivity variation of the emulsion with wave length. This is, however, a much simpler process than is ordinarily realized.

2. *Photometry.* a. *Homochromatic Spectrophotometry.* We wish to compare the intensities of two spectral lines of nearly the same wave length, or of two beams of light of the same color, which may be pure or a mixture of wave lengths provided both are of the same quality. We photograph the two simultaneously, or, if they are under control, successively, on the same plate, for the same length of time. They are sensibly identical except for intensity, and we use the densities which they produce to determine their relative intensities. To do this we subject the plate to a series of equally long exposures to light of the proper quality and of various intensities differing by known amounts. After measuring the plate we can construct the characteristic curve obtained from these exposures, and from it we can determine by interpolation the intensities corresponding to the density values due to the lines in question.

Where the contrast is not changing rapidly with wave length the same characteristic curve can often be used for lines differing by as much as 100 A. As indicated in Fig. 4, there are certain portions of the visible and near ultraviolet where the contrast changes quite slowly, especially in emulsions of low or medium contrast. It should be borne in mind, however, that constancy in contrast is not sufficient; the characteristic curves for the wave lengths used must be identical in shape. With determinations of absorption and reflection coefficients the beams compared are identical in wave length and the determination of the proper characteristic curves is all that is necessary.

One is apt to fail to note that it is quite permissible to apply the calibration exposures to the plate with a different spectrograph than

that used for the spectra to be measured. Also, as in several methods to be discussed below, the unknown lines can be made to impress their own calibration exposures on the plate. But intermittent calibration exposures must not be used when the source to be measured is constant, or vice versa.

Where spectral lines are to be compared it frequently happens that the spectrogram shows a continuous background (apart from that due to fog) superposed on the lines. The density measured over the lines is then that due to the sum of the intensities of line and background, and the density of the background must be measured near the line. This and the density over the line are to be converted into intensities by means of the characteristic curve, and the background intensity is then directly subtracted. When the background intensity is larger than about ten per cent of that of the line the errors of photometry increase rapidly as it increases, especially if the background correction is at all variable. In the case of calibration exposures the background can usually be ignored, and in any case should not be subtracted from the line intensities since its effect is present in all of the densities used in an amount proportional to the intensity of the exposure, and it has the proper wave length (provided of course it is not due to general scattering in the spectrograph system, when it must be eliminated.) But it is desirable, on account of the Eberhard effect discussed below, to have the calibrations of approximately the same size and shape as the unknown lines (see Fig. 5), as in some cases where a strong background exists it may produce errors from this cause. However, the error in this case is due to the presence of the background *near* the line rather than *under* it, and it can usually be removed by the interposition of screens.

b. *Heterochromatic Spectrophotometry.* In any particular piece of heterochromatic photometry it is neither necessary nor sufficient to determine the variation of emulsion sensitivity with wave length separately, since what is wanted is the variation in what may be termed the total or over-all transmission sensitivity of the spectrographic outfit. This will be influenced not only by the variation of emulsion sensitivity, but by the variation with wave length of the transmission or reflection of the prism or grating used to produce the spectrum; by the contolling apertures of the system for different wave lengths; by the dispersion variation of the spectrograph; by the characteristics of any lenses in the system; by the astigmatism of the system, and by several other factors. While all but the first remain sensibly constant from one experiment to the next, it is generally convenient to lump them all together by using

a null method. Before or after the source to be measured is photographed, a standard source of known energy distribution is photographed with the same time of exposure. The densities of known and unknown are then converted into relative intensities J at each wave length by means of the proper calibration curves, and since the (so-called) absolute intensity I of the standard source is available, that for

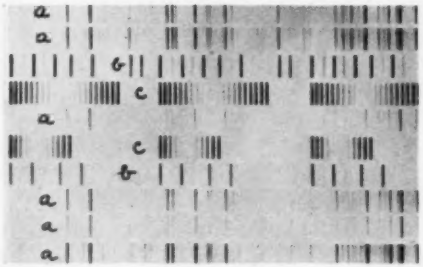

FIG. 5. *A portion of a spectrum plate obtained in making intensity measurements with a 10 meter concave grating in the second order, dispersion 0.79A/mm. (a) Spectra to be measured, (b) Exposures to a standard incandescent lamp at various temperatures, (c) Calibration exposures. The exposures (b) and (c) are taken through narrow slots in a diaphragm, and occur in sets 25A apart, taken simultaneously, the plate being moved sideways through about an inch during the taking of a number of sets of exposures. The lines in the exposures a in this particular specimen are too uneven to be of value for photometry. Five exposures of type a present on the original plate are not shown in the figure.*

the unknown can be determined by a simple interpolation process. It is evident that the function of the standard source is to furnish data to tie together the various calibration curves at the different wave lengths involved. This must be done in most cases where the lines being compared differ by more than 20A.

III. GENERAL APPARATUS AND METHODS

The chief pieces of apparatus required to carry out photographic spectrophotometry are the following:

(1) A spectrograph to produce the exposures, including a device to focus light on its entrance slit.

(2) A shutter which can be kept open for a definite time which will not vary by more than about 0.5%.

(3) A source of light for calibration, which will cover the same wave-length region and be at least as intense at certain suitably distributed wave lengths as the source to be studied. It must have a constant

average value in time, and the same intermittency characteristics as the unknown source.

(4) A means of varying the intensity of this source at each wave length by a known amount. So called neutral methods vary the intensity by the same amount at all wave lengths; this is convenient, but not necessary.

(5) A standard source of known energy distribution covering the same wave-length range as the source to be studied. (This is not needed for homochromatic photometry).

(6) A controllable and even means of developing, fixing, washing, and drying plates or films. Reproducibility is desirable but not necessary.

(7) A densitometer or microphotometer for measuring the blackening of the plates.

Each of these requirements will now be discussed in detail.

1. *Spectrographic Outfits.* a. *The Spectrograph.* The choice of the spectrograph to be used will depend greatly on the type of work to be undertaken. Both prism and grating instruments are satisfactory; any spectrum that can be produced can be measured photographically. Where extremely weak light sources are to be measured prism instruments are preferable, since their light transmitting power is so high that one may work with short exposures, which is very desirable in photometric work. Exposure times between 10 seconds and 100 seconds are the most convenient, but by no means requisite; one cannot fix definite limits, since exposures of less than 10^{-5} seconds and longer than a week may be perfectly measurable.

The most valuable single item in photographic photometry is the space on an individual plate, so it is desirable to use low dispersion for the sake of accuracy and economy of effort. On the other hand, high dispersion reduces the intensity of the continuous background relatively to that of the lines, and this is highly desirable. The compromise to be selected depends entirely on the character of the work. For intensity measurements in complex spectra such as titanium or vanadium I have used the second order of a 10 meter concave grating, with a dispersion of 0.73 A per mm. Here the separation of close lines and the reduction of background are the most important requirements. For less complex spectra like those of calcium and copper the first order of a 21 foot concave grating of 15,000 lines to the inch is very satisfactory, with a dispersion of about 2.57 A per mm. For spectra as simple as zinc, cadmium, and silver the first order of a one meter grating of dis-

persion about 16 A per mm is suitable. Much work has been done on the alkali metals and helium with prism apparatus, the dispersion varying from 10 to 50 A per mm in the visible to values four to ten times as great in the ultraviolet.

For absorption or reflection work where a continuous spectrum is desired it is practically essential to use a prism instrument, unless special precautions are taken. Scattered light is much stronger with a grating than with a good prism, and becomes notably intense when a continuous spectrum is used. The overlapping of orders in certain regions is often a drawback of the grating, although it can be controlled by suitable filters.

In connection with the question of the dispersion to be chosen, it is generally necessary to use a fairly wide slit for line spectrum work, in order to have flat-topped lines to measure. Otherwise, where lines of different shapes must be compared, it is necessary to make a large number of density measurements to determine the shape of each line, the resulting intensities being integrated to give the total intensity. If, however, the slit is so wide that the image contains every portion of the line superposed on every other part, there will be a flat top; and a single intensity value obtained from the height of the flat top will be proportional to the total intensity of the line. The necessary slit width is somewhat more than that of the widest line being measured (in the case of symmetrical lines a width one-half as great would do). The flat-topped portion of the line should be wide enough to cover the light beam of the microphotometer. The objections to still further widening of the slit are that the continuous background is increased relatively to the lines, and resolving power is lost.

For many methods of photometry an astigmatic spectrograph is a decided advantage; for others a stimatic optical system is required. Often the type of photometric method to be used will be decided by the properties of the spectrograph system available. Evenness of illumination along a spectrum line is a prime requisite, and astigmatism aids in securing it.

The spectrograph should produce straight rather than curved spectrum lines. Each line to be measured should be at least 3 mm long if possible, as the effect of graininess and other irregularities in the emulsion is apt to be noted where a shorter length is used.

Mountings of the Littrow type are often undesirable for photometric work unless the lens can be slightly tilted to avoid reflection of the light from the surfaces of the lens back on to the plate. Scattered and reflected

light must be held to a minimum. Requirements for special spectral regions will be mentioned below.

b. *Grating Mountings.* Where a concave grating is used, a modification of the Paschen mounting is probably most suitable for general photometry. The Rowland and Eagle[15] mountings suffer from the disadvantage that only a limited region of the spectrum can be photographed at one exposure, so that lines differing greatly in wave length cannot be directly compared. When a stigmatic spectrum is required the Wadsworth mounting is excellent.[16] This cuts the dispersion in half, but increases the brightness of the spectrum, especially in the higher orders. It is probably the best high dispersion mounting for use with a neutral wedge or a step-weakener, when these must be placed at the slit.

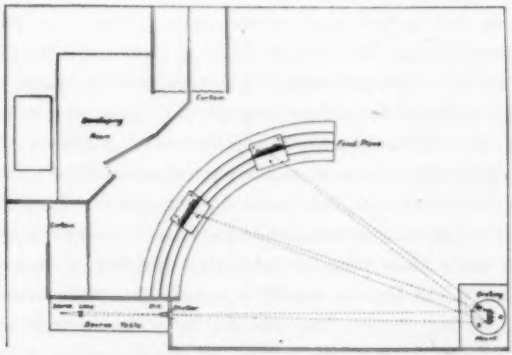

FIG. 6. *A convenient type of grating mounting for use in intensity work, suitable for gratings of any radius from one to ten meters or more.*

In the Paschen-type mounting the slit, grating, and focal plane are fixed and one can distribute plates practically continuously along the spectrum. Fig. 6 shows a mounting of this type as used in the Stanford laboratory, 10 meter, 21 ft., and 1 meter gratings having been mounted in this manner. As many as twelve cassettes are frequently used simultaneously, enabling one to compare numerous lines in a large spectrum range under identical conditions of excitation. A still better mounting would be a double one of the type shown in Fig. 6, so that orders on both sides of the central image could be used simultaneously. The cassettes on the right hand side could then be arranged to cover the gaps in the spectrum between those on the left, as ordinarily the construction of the cassettes requires that a space one-fourth to one-third

the length of a plate be left between adjacent plates. With large gratings it is seldom possible to find a room of sufficient size to house such a double mounting, however.

Any mounting of high dispersion where only a single plate can be exposed at a time will be found very wasteful of effort, and it is of course impossible to compare widely separated spectral regions directly with such a mounting, except where orders happen to overlap suitably.

c. *Illumination of the Spectrograph Slit.* As most methods of photographic photometry require the insertion of light weakeners in the beam before it enters the slit of the spectrograph, it is necessary to focus the light on the slit by means of a lens or mirror. It is generally desirable that a magnified image of the source be used to increase the uniformity of illumination of a fairly long slit, and to allow easier control of a flickering source. Ordinarily magifications of four to ten-fold are useful. Care should be taken that the effective aperture of the lens or mirror be slightly greater than the controling aperture of the spectrograph.

As the standard lamp and the source being studied must be substituted for one another and photographed under identical conditions, care must be taken that they are placed in identical positions in front of the lens. Otherwise, especially in the case of a lens with appreciable chromatic aberration, the ratio of the amounts of light entering the slit from the two sources may depend on wave length, and large errors may ensue. For this reason it is desirable to use a corrected lens so that a small error in placing one or the other of the sources will produce only a second order disturbance. Very excellent quartz-fluorite and quartz-rocksalt objectives of 25 mm aperture and 250 mm focus can now be obtained, the focal length in some cases varying not more than a few millimeters over the whole visible and ultraviolet regions. A focal length of between 150 and 300 mm is most useful.

Mirrors cause no chromatic difficulties, but they are very wasteful of light in the ultraviolet and are more cumbersome to handle. Their astigmatic properties are frequently useful in producing more uniform illumination of the slit. Silicon, chromium, and nickel are good materials for mirrors to be used with ultraviolet light; silver is very poor for the purpose.

d. *Plate Holders and Cassettes.* Any commercial spectrograph can be used for intensity work; but when apparatus is being designed or modified for this particular use, certain features should be incorporated which often are not possessed by apparatus designed chiefly for determinations of wave length. We bend every effort towards getting as

many records of densities on a single plate as we possibly can, knowing that once these are converted en bloc into values of intensity the action of the plate may be forgotten. Plates which are large enough for only two or three spectrum photographs are wasteful of effort. At least fifteen spectrum photographs can be obtained on 4×10 inch plates, a standard size in which almost any emulsion can be obtained. In the Stanford laboratory eight spectrographs, ranging from a 10 meter concave grating to a small quartz prism outfit have been rebuilt, when necessary, to carry this standard size of plate. Much time is thereby saved, as the same plate can be subjected to successive exposures for calibration or other purposes in different instruments. In any case, the plates used should be as large as is convenient.

Any ordinary type of plateholder can be used, but the cassette in which the plateholder is held may well contain certain special features. Thus two complete sets of calibration exposures at all the necessary wave lengths can be impressed in the space usually required for a single spectrum by a method which has recently been described in this journal,[17] whereas ordinary methods would involve the use of two whole plates to give the same data. Fig. 5 shows a portion of a typical plate.

Where optical wedges or step-weakeners are used in the focal plane it is often necessary to have long spectral lines, broken up into sections of different density. The proper changes in cassettes to make them suitable for these cases are obvious.

2. *The Timing of Exposures.* A stopwatch and hand-operated shutter are satisfactory for exposures of not less than 20 seconds when calibrating. For exposures to the spectrum under measurement and to the standard lamp exactness of timing is not required, provided that the exposures to all parts of the spectrum are begun and ended simultaneously. For routine laboratory work a clock delivering an electric impulse every second is most desirable. This can be constructed if necessary from almost any cheap type of pendulum clock, the period (and the time keeping qualities) of this being altered by substitution of a shorter or longer pendulum to make the period approximately one or two seconds. The pendulum is made to close an electric circuit every second in the usual manner, and operates a magnetic shutter on the spectrograph whenever a key is held closed. A very poor clock if properly handled can be made to give impulses uniform to less than 0.1 second, which is good enough for exposures of 10 seconds or over.

In the case of exposures much greater than 10 seconds it is convenient to have an electric counting device which will close a key at the end of a

predetermined time. Such arrangements are in very common use now, and need no detailed description. A convenient type is made from a ratchet wheel having 100 teeth, this being driven forward one tooth at a time by the action of a spring which is released once each second by an impulse from the clock. It is desirable that the spring rather than the magnetic impulse should turn the wheel, while some simple device should be provided to prevent more than one tooth passing the pawl at a time. Pins placed around the periphery of the wheel can be inserted in suitable holes to close a contact so that exposures between 1 and 100 seconds can be automatically made.

The electromagnetic shutter in the spectrograph can be of any type with positive action, but in general it should be mounted behind the slit rather than in front, so that adjustment of the light beam or of filters can be made conveniently. An iris type shutter of fairly large aperture is very convenient. The self-setting type should be used, and can be readily operated by the clapper of an ordinary electric door bell with the vibrating arrangement disconnected.

3. *Sources of Light for Calibration.* The source of light being measured need not be absolutely steady. But if it is an ordinary arc, it should be kept as steady as possible, and the calibration source should be very constant; while if it is a spark, the calibration and standardization sources should be uniform and controlled spark discharges of the same type, or constant sources made intermittent in the same degree.[17a] It is necessary that the calibration source should cover the same wave-length range as the source being studied, delivering light of frequencies in the neighborhood of those in the latter, although not necessarily exactly the same; that it should show no intermittency different from that of the source being studied; and that it should remain constant in average intensity. The last requirement is only essential when a number of successive calibration exposures are required; where all are taken simultaneously as with the wedge or step-weakener (see below) a source of variable intensity can be used, and in this case it is generally desirable to use the source being studied itself as the calibrating source. It is of course not necessary to know the intensity distribution in the calibrating source.

In the visible and infrared regions an incandescent lamp run from a storage battery makes a most convenient source for calibration. Such a lamp with a quartz window can be used in the ultraviolet, although the intensity is usually rather low below 3000A unless the calibrating is being done in a low dispersion instrument. When continuous sources

are being used for calibrating plates exposed to line sources it is desirable to produce artificial lines on the plate by means of diaphragms, as in Fig. 6, to reduce errors due to the Eberhard effect.

In the region 4350–2400A the mercury arc[18] in quartz makes a convenient calibration source, when operated from a storage battery with reactance and resistance in series. Great care should be taken to ensure that the arc attain a final steady state, as there is apt to be a tendency for a drift in intensity. If both voltage across and current through the arc remain constant, so also does its radiation, except for very slow changes in the transparency of the quartz envelope. But in order to control current and voltage separately it is necessary to be able to control the ventilation of the arc.

Although the mercury lines are fairly widely separated, they are so spaced that calibration curves for the range given above can be readily interpolated for any desired wave length. As the contrast and shape of the curve are all that are desired, and as these change rather slowly in the region referred to, calibration curves interpolated every one or two hundred angstroms are sufficient. With very high dispersion outfits all the calibration curves on a single plate will generally be found to appear the same, except in the regions near 5000A and 2500A.

When an intermittent source such as the spark or a discharge tube run on ac is being studied, the most convenient calibration source is generally either the unknown or a source like it having a constant average intensity in time, but with the same intermittent characteristics. A spark between rotating wheel electrodes[19,20] can be made to fulfil this requirement; its constancy can be watched by focusing its total radiation on a thermocouple connected to a galvanometer. Care should be taken that the calibration spark and spark being studied have the same number of discharges per cycle; this can be checked by observation with a shaking or rotating mirror. A change in tone of the spark during an exposure is undesirable, as it is the sign of a change in discharge frequency and in intensity. Unless it is fed by a motor-generator set run from a constant supply the spark transformer voltage should be regulated.

Below 2400A a hydrogen discharge tube run on steady dc is probably the most convenient calibration source, and it provides a good continuous background extending far into the ultraviolet.[20a] Special control of pressure and other conditions is required to insure steadiness. Steady sparks can also be used in this region.

4. *Methods of Varying Intensity for Producing Calibration Curves.* These may be classified as *neutral* and *selective* methods. The former have the advantage that the transmission values at all wave lengths can be determined by making measurements with any mixture of wave lengths, while the latter must be calibrated near every wave length at which they will be used.

a. *Neutral Methods.* (1) Diaphragms. If in the optical system there is a beam of light having the same cross-section for each of the constituent wave lengths, and uniformly intense over that cross-section, diaphragms of different apertures can be introduced to cut down the light by known amounts, provided always that the spectrograph does not vary in transmission from point to point of its aperture. This is one of the simplest methods, but the conditions are very difficult to fulfil accurately; especially with gratings, where the diffracting surface may be uneven in reflection. With prism-lens combinations radial sector diaphragms are generally better than zonal diaphragms.

(2) Perforated Screens.[20b] These are in reality diaphragms, so constructed that the foregoing requirements do not hold. Each small hole in the screen acts as a diaphragm, and the transmission of the screen can be measured or in rare cases computed. Certain simple precautions must be observed in the use of screens to avoid errors due to diffraction. I have discussed these at length in a recent paper.[21] The screens are not additive unless they are rocked. Woven wire screens are in general more uniform than those made by punching holes in metal sheets.

(3) Variable Slit Width. When continuous sources are used for calibration it is often convenient to vary the intensity by changing the slit width.[22] Usually only four or five values of intensity can be obtained in this way for a single set of lamp conditions. By changing the brightness of the source four or five new points can be obtained, and the two sets joined together where they overlap, to give a continuous curve.

(4) "Rasters."[23] These are ingeniously constructed diaphragms arranged to combine the advantages of the wire screen and the step-weakener. They appear to be truly neutral, but can be used only in a stigmatic mounting, unless a special type of astigmatic mounting described by Frerichs is available; and this latter can only be used for one plate at a time. Under such circumstances they appear to be equal in value to the step-weakener, but care must be taken to see that they are uniformly illuminated.

(5) Rotating Sectored Disks.[24] In most work these had better be avoided, as they produce intermittent exposures and vary not the intensity but the time of exposure, thus introducing both intermittency and reciprocity errors. In homochromatic photometry they are frequently used in certain special methods later to be described, but the conditions are there so chosen that the two sources of error apparently tend to partially cancel. This is not true in most cases, however.

(6) Variation of Time of Exposure. In cases where the reciprocity law holds it is legitimate to use H. & D. curves to determine the characteristic curves of the emulsion. This method has been much abused in the past, however, and any investigation in which it is employed and not specifically verified is open to suspicion. If one takes a small amount of trouble at the start, it is thereafter easier and quicker to vary the intensity than the time; and since in any case the intensity must be varied in order to test the reciprocity law, the apparent advantages of varying the time become scarcely noticeable. The method is one for the expert rather than for the beginner, and the former seldom uses it.

(7) Adoption of Theoretical Line Ratios. The ratios of the intensities of spectral lines in multiplets are known to be atomic properties and often have simple integral values. In a quintet multiplet, for example, we may have twelve lines fairly uniformly distributed over an intensity range of several hundred. In the case of multiplets which have been found experimentally to have normal intensities there is no reason why these lines should not be used for producing calibration curves. It is necessary, however, to choose narrow multiplets which have a single or several very close upper states, (unless the conditions of excitation are definitely known) and to be sure that no self-reversal is present. This last condition is often very difficult to fulfil, and the method should not be used except in connection with a more accurate one for quantitative work.

b. *Selective Methods.* (1) "Neutral" Filters. It seems a contradiction in terms to include "neutral" filters under this heading, but the fact remains that even the best materials are neutral over limited spectral ranges only, and it is necessary in any particular case to prove this neutrality, in doing which one often loses much of the advantage of the method. Very good approximations can be found for portions of the visible, but the commercial neutral gray glass or gelatine filters are apt to vary more than 5% in transmission in a comparatively limited range. Platinum sputtered on quartz[25] has been found to be fairly

nearly neutral through much of the visible and ultraviolet. Professor Ornstein informs me that in his institute antimony has lately been found to be superior to platinum; it is evaporated in vacuum and condensed upon very clean quartz plates. All of these materials should have their transmission measured at various wave lengths, and the necessary correction factors should always be applied. Filters are superior to wire screens in that they need not be rocked, can be fashioned into wedges and step-weakeners, and entail no diffraction errors. They are generally not so uniform as screens, however, and sometimes introduce refraction errors; for commercial filters of glass are often slightly thinner at one edge than the other, causing a prism effect which may throw the light slightly off the slit. Also changes in focus differing for different wave lengths may occur when filters are set across the beam if they are fairly thick.

(2) Polarizers. Crossed Nicol Prisms can be used in the visible, or Foucoult prisms in the ultraviolet, the intensity of the light transmitted varying in either case with the angle of rotation of the prisms. One is justified in suspecting that they may introduce appreciable errors due to the polarization of the light, since the reflecting power of a mirror or concave grating depends on the polarization of the incident light. They are much more commonly used in visual than in photographic methods, and possess few advantages for the latter.

(3) Non-Neutral Filters.[26] Often it is desirable to weaken one color of the calibrating light relative to another in order to approximate more closely the intensity distribution of the unknown source. Selective filters are useful for this purpose, and are no more difficult to calibrate for transmission than the so called neutral filters. The catalog of Wratten Filters published by the Eastman Kodak Co. and the lists of filters given in Vol. 5 of the I. C. T. are very useful for the selection of proper filters for this purpose.

(4) Theoretical Line Shapes. Ornstein and Minnaert[27] have made use of the shapes of spectral lines produced by a spectrograph to give the density-intensity relations of certain uncalibrated plates. This method could be extended in various ways, and is very useful under certain circumstances, but must be used with circumspection.

(5) Variation of Temperature of a Source. Laws relating the brightness of a radiator to its absolute temperature can be used for producing calibration marks, but the method is rather indirect and cumbersome.

(6) Special Methods. These include wedges, step-weakeners, coarse ruled gratings, selective reflection, etc. The more important ones will be discussed below in connection with the special photometric methods in which they are involved.

5. *Standard Sources.* a. *Continuous Sources.* The black body is the source of which the intensity distribution is most accurately known. It can be used directly as a standard, or indirectly by calibrating from it secondary standards such as an incandescent ribbon or spiral of tungsten. The energy distribution from a black body is given by Planck's Law. For values of λT greater than 3000 (λ being evaluated in cm) the simpler approximation form known as Wien's Law is adequate for our purposes, viz:[28] $Id\lambda = C\lambda^{-5}e^{-hv/kT}d\lambda$. This can be written in the form $\log_{10}I = 26 - 5\ \log_{10}\lambda - \text{antilog}\ (7.7939 - \log_{10}\lambda - \log_{10}T)$ which is somewhat more convenient for use in computations on a calculating machine. Tables of certain black body emission values are given in Vol. 5 of the I.C.T.

In practice black body radiation is simulated closely enough by radiation from a small hole in a heated enclosure of almost any material of sufficiently high melting point, or from narrow slots in carbon rods or tubes. Frerichs[29] describes a black body formed of a hollow carbon tube with central plug heated in a platinum resistance furnace to 1400°A. In the Stanford laboratory a six mm graphite rod about 20 cm long is heated in a partial vacuum to a temperature of 2200–2800°A by a current of 200–300 amperes from a 10 kw step-down transformer. The light used is taken from a notch 1 mm wide by 3 mm deep and 3 cm long, and passes out through the quartz window Q waxed to the water-cooled metallic jacket surrounding the filament as shown in Fig. 7. A single rod lasts a number of hours, and can be replaced in a few minutes. The temperature is measured by means of an optical pyrometer. A correction of about 20° must be applied to the observed temperature to allow for absorption of light by the window; its exact amount can be determined by inserting another similar window in the beam.[30] Dorgelo[31] describes other types of black body.

If a black body is not available one may use the radiation from tungsten, carbon, or other filaments, applying corrections for the departure from black body conditions. If the temperature of such a radiator is measured with an optical pyrometer of the disappearing filament type,[32] the so-called *brightness temperature* T_b at a certain wave length (generally 6650A in the red) is obtained. For several materials we know the relation between this and the *color temperature* T_c, which

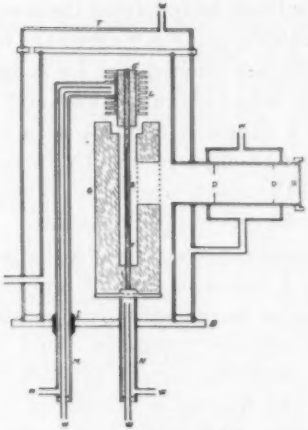

FIG. 7. *A convenient black body furnace for use as an intensity distribution standard throughout the visible and into the ultraviolet as far as 2600A. Q, quartz window through which light from the slit S in the filament F emerges. A 10 kw transformer, terminal voltage 10 volts, sends a current of from 200 to 300 amperes from M to N. The top T is waxed to the base B with soft laboratory wax, and is exhausted to a few mm pressure through P. Some water vapor sweats through the brass jacket (which should preferably be made of copper) but this is largely taken up by the graphite cylinder G which gets red hot. The 1/4 inch filament F can be run for a number of hours at temperatures up to 2800°A, but is most useful when run at about 2500°A. A constant stream of water passes through the cooling system by means of the pipes marked W. A new filament can be inserted in a few moments when one burns out.*

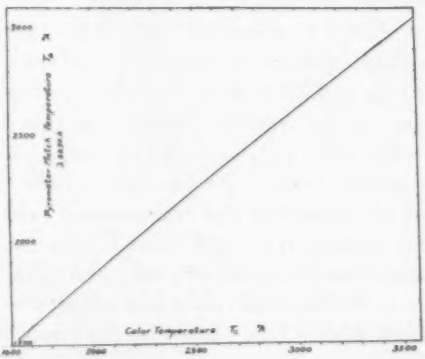

FIG. 8. *Graph showing the relation of the color temperature of a tungsten strip to its brightness temperature as measured with an optical pyrometer with red filter (brightness match at wave length 6650A). The energy distribution of the light from the strip in the visible region can then be calculated from Wien's law, substituting for T in that formula the value of T_c obtained from this curve.*

is the temperature of a black body having the same energy distribution over the visible spectrum range. Fig. 8 shows the relation between color temperature and brightness temperature for tungsten, the data being taken from that of the Nela Laboratory workers[33] and Zwikker.[34] It is thus possible to use a tungsten filament as a standard source, the energy distribution being computed directly from Wien's Law with T_c substituted in place of the true temperature. This is probably the most convenient source for routine use in the visible.

A standard type of lamp for this purpose is that containing a tungsten ribbon approximately 1 mm wide and 25 mm long, taking about 20 amps at 5 volts. This can be run with great constancy from a bank of say six 6 volt storage batteries in parallel. Such lamps, calibrated for color temperature in terms of current through the filament, can be obtained from either the Bureau of Standards or the Nela Research Laboratory. In the range 4500–3600A a correction to the energy distribution curves should be made on account of the selective transmission of the glass envelope. This transmission can be measured for various wave lengths in a spectro-radiometer.[30]

Such lamps can be fitted with a quartz window,[35] or indeed with a complete fused quartz bulb. Those of the latter type supplied by the Philips Glowlamp Works, Eindhoven, Holland, are designed to be burned in an inverted position, so that the evaporating tungsten, being carried upward by the convection currents in the gas contained in the lamp, will be deposited where it will do no harm. Their usefulness in the ultraviolet below 3600A is somewhat limited by the paucity of data on the emission characteristics of tungsten in this region. Even up to wave lengths as long as 4500 a small emissivity correction is desirable, which is, however, in the opposite direction to that required for the varying transmission when a glass envelope is used. Lamps with quartz windows can be readily compared with a black body by photographic photometry, and all emissivity and transmission errors can be eliminated by a single correction at each wave length. v. Alphen[36] notes that the use of light from the center of a tungsten spiral has advantages in the ultraviolet, as the internal reflections apparently compensate for the deviations from Wien's Law caused by the emissivity variation of tungsten in this region.

In certain cases it is desirable to determine the actual energy distribution from an incandescent source by means of a thermoelectric device. The double monochromator of van Cittert[37] is most convenient

in such cases, as no parts need be moved except an adjustable slit. For the highest accuracy it should be provided with color-corrected lenses to avoid changes in aperture with wave length. An ordinary single prism monochromator will serve, provided the complete aperture of the system is not used so that the effective aperture can be kept constant by means of external diaphragms while the lenses are being refocused for various wave lengths. The fixed-prism type[18] sometimes has advantages over the movable-prism type[38] where the losses of light by reflection may be very great in certain regions; commercial quartz monochromators are obtainable, however, which avoid this difficulty.[39]

Vacuum thermo elements of very high sensitivity and quick action, of the new type designed by Moll and Burger,[1] can be obtained commercially; in glass, uviol glass, or quartz envelopes permanently evacuated, or in envelopes provided with a fluorite window, which require the use of a pump. When used in connection with a thermo-relay[2] their sensitivity is limited only by the effect of the Brownian movement on the galvanometer. These, when used in conjunction with a good monochromator of large aperture leave little to be desired for standardizing lamp sources.

As the energy distribution at the light source is what is required, it is necessary to correct for the transmission of the monochromator, which when used in this way is termed a *spectro-radiometer*. Several ways of measuring the transmission of this at various wave lengths are available,[38,17] of which those involving the use of another monochromator and a photoelectric cell are probable the simplest. The use of any fixed arm monochromator with a thermocouple directly in front of the main spectrograph, in the manner described by Dorgelo[40] for the van Cittert instrument, is permissible and avoids measurement of the transmission of the monochromator, provided that the aperture of the beam striking the thermoelement and entering the spectrograph is kept constant.

Various other continuous sources whose energy distributions have been determined under certain conditions are listed by Dorgelo,[5] but their usefulness is limited to certain special cases.

b. *Line Sources.* Where available, constant continuous sources are most desirable for standardization except in certain cases of use with grating spectrographs where scattering of light causes trouble. Their spectra can be blocked off by means of diaphragms to give artificial lines, as was shown in Fig. 5. In the ultraviolet region the radiation from most materials at temperatures obtainable in the laboratory is too

weak for use with high dispersion equipment below about 3000A, (2300A has been reached with low dispersion prisms), and it is necessary to turn to line sources for standardization purposes. None with known intensity distribution is at present available, so this must be measured in each case with a spectro-radiometer in the manner described under 5a.

Practically the only source which has been used in this way up to the present is the quartz mercury arc.[41] It possesses relatively few lines and these are of great strength, and it can be controlled in such a manner that it will be very steady. The lines are on the average close enough together so that two or three can generally be impressed on each plate even with high dispersion apparatus, and they are far enough apart so that they can be resolved with a small prism monochromator. Each individual arc must be separately measured, and the measurements must be repeated for each set of running conditions.[18] For this reason one or two definite sets of conditions are generally chosen for all standardization work in a given laboratory.

As the arc warms up its terminal voltage rises and the current through it decreases. The relative value of these two quantities depends on the arc temperature, so it is necessary to provide a means of varying the ventilation of the arc to control this. The methods of controlling mercury arcs are described in detail in the articles cited. The mercury spectrum is useful for standardization only between 4360A and about 2500A, since the lines at longer and shorter wave lengths are too widely spaced or too weak to be of general use.

There seems to be no doubt that cadmium arcs[42] or arcs of other metals could be used to supplement the mercury arc, while hydrogen discharge tubes carrying high currents can be used in the ultraviolet and extreme ultraviolet. There is a great need for further sources to be used as standards of energy distribution; any constant continuous or line source which can be run under controlled conditions and which is of sufficient intensity for measurements with the spectro-radiometer can be made to serve.

c. *Dispersion Corrections.* If we have two practically monochromatic lines of the same shape produced by a spectrograph and the spectrograph slit is widened sufficiently they will have flat tops and equal breadths, provided the spectrograph has uniform dispersion, or if it has a "true" variation in dispersion such as that caused by the varying dispersion of a prism, which can have no effect on monochromatic lines. But if, as is the case in many instruments, the photographic plate is

struck at different angles by beams of different wave lengths, or if the spectrograph produces larger images of the slit with long wave lengths than with shorter ones, say, there will be a difference in the breadths of the two lines, and a width correction must be applied before the heights of the lines can be considered proportional to their areas. Now if we consider two regions of width $d\lambda$ taken from a continuous source, the same effects would be observed, but in addition an effect would occur because of the overlapping of neighboring slit images, this varying with the "true" dispersion. Hence if we compare an unknown line source with a continuous standard source, or vice versa, part of the effect of the "apparent" ("true" plus "false") dispersion variation will cancel out, while the effect due to the "true" dispersion variation will not. Thus in the case of a grating mounted in the Paschen manner the dispersion variation is caused by the change of angle at which the light strikes the plate, and by changing focal distance ("false" dispersion variation) and no dispersion correction need be made, even when line and continuous sources are being compared. But in the case of a prism instrument it appears that it is necessary to distinguish between the "true" and the "false" dispersion variation present, and to correct for the first of these rather than for the "apparent" variation of dispersion. The same result can of course be achieved by correcting for "apparent" variation of dispersion, and then applying a correction for changing line breadths. In the Fery spectrograph or in the ordinary quartz prism instrument a large amount of "false" dispersion variation is usually present due to the variation of the focal lengths of the lenses with wave length, and to the curvature of the focal plane. The separation of these two effects is not difficult, but appears to be often overlooked. In any photometric investigation the possible necessity for corrections of this sort, due to differences between the source being studied and the standard source, should be carefully gone into.

6. *Development.* a. *Developers.* Either plates or films can be used satisfactorily for photometry, but plates are somewhat more convenient to handle, and can be used in most cases, since very exact focusing is not required for intensity measurements, and it is seldom necessary to curve the emulsion except with low dispersion apparatus. It is desirable to use no part of a plate within half an inch of any edge, and where possible, to cut plates from those larger than needed, as the emulsion is apt to be more uniform in the center of any given coated area. Seasoned plates which have been stored on edge in a cool place give best results.

The choice of developer is rather important, since very uniform development without staining of the emulsion is wanted. The two developers which seem to be in most common use for photometry are elon (or metol)-hydrochinon, and rodinal (1 part to 20 parts water). A good formula for the former is:

Distilled water	10 l
Elon (or metol)	14 g
Sodium Sulphite	212 g
Hydrochinon	56 g
Potassium Bromide	150 g
Sodium Carbonate	300 g

This can be mixed in large quantities and will keep a number of months in stoppered bottles. When used it is to be diluted with two parts of distilled water to one of developer. At a temperature of 18°C the proper development time for this formula is about five minutes. The rodinal developer need only be mixed with water at the time of use. Developers such as pyro and glycin should be avoided.

b. *The Eberhard Effect.* If a plate be uniformly exposed to light of moderate intensity, and a small area be then given an additional heavy exposure, on development a region of abnormally lowered density will be noted around the dense portion. This is due to what is known as the *Eberhard effect*, and is caused by the fact that when a heavily exposed area of emulsion is being developed a large quantity of soluble bromide is set free which acts as a restrainer and slows up the development of surrounding regions. Numerous devices have been proposed to reduce this effect, the most useful being (1) Selection of a developer producing small Eberhard effect (elon-hydrochinon and rodinal are good in this respect); (2) Use of moderately complete development, stopping only when appreciable fog appears; (3) Constant positive agitation of the developer to distribute the reaction products more uniformly over the emulsion; (4) Use on the emulsion of exposed regions all of approximately the same shape and of uniform distribution. The last condition is of course difficult of exact realization, but it can be approached, as mentioned above, by breaking up a standard continuous spectrum into artificial lines when comparing it with a line spectrum.

Dobson, Griffith and Harrison[6] describe various methods of producing uniform development, their plunger developing apparatus or the mechanical rocking and brushing outfit of Bloch and Horton[43] being

probably the most satisfactory. If the plates are constantly brushed during development[44] and the other precautions listed are taken very little trouble due to Eberhard effect is found. In the design of developing machines the important point is that turbulent motion is required rather than streamline flow, no matter how rapid.

c. *Treatment after Development.* In fixing, washing and drying plates the usual precautions observed in ordinary photography are sufficient. A good hardener should be used to prevent marring of the emulsion; formulas for these are given by manufacturers in practically all boxes of photographic material. After washing, the plates should be carefully swabbed on both sides a number of times (with a tuft of wet cotton, or better, the palm of the hand) to remove the "bloom," and then rinsed in distilled water so that no drying marks or dust spots will remain. Slow drying is preferable to the use of a fan.

It is a common misapprehension that when a number of plates which contain spectral lines which are to be compared have been exposed simultaneously, great care must be taken that they all be given the same development. As a matter of fact the plates may of different emulsions and may be given entirely different treatment on development. This is because each plate is separately reduced to intens ties, the exposures to the standard source furnishing the bases for comparison.

d. *The "Brush Effect."* Reference is sometimes made in the literature to the statement[45] that the latent image may change fairly rapidly after exposure, so that two exposures on the same plate should not be compared if one is made immediately before development while the other has been made at some previous time. Careful experiments[46] have indicated that no such effect exists, except possibly within two or three seconds after exposure. It is a good plan, however, to take unknown, standardization, and calibration exposures as near together in time as possible, since variations in humidity, temperature, etc. can undoubtedly affect the characteristics of the emulsion to some extent.

7. Densitometers and Microphotometers. a. *General.* After the various exposures have produced their records on the plate these must be measured by having their densities compared so that the corresponding intensities can be determined. Many designs of microphotometer have been suggested for this purpose, but only objective types will be discussed here. Subjective microphotometers in which the eye is used as a balancing instrument have been brought to a

high degree of development, but for measuring photographic plates they have been practically entirely displaced by those using the thermoelectric or photoelectric methods. They can be used where the accuracy of a single reading need not be over 5%, but are fatiguing and slow compared to the newer methods. Various types of visual microphotometer are described in Kayser's *Handbuch*, and by Dorgelo[5] in his review of photometric methods. For the measurement of extremely high densities their use is sometimes necessary, and an instrument especially designed for this purpose has been described by Jones.[46a]

Practically all microphotometers except those of the null type can be used either with a telescope to measure deflections directly or with a recording device which impresses a record of these on photographic paper or plates. The latter device is of great service in determining line shapes or the general contour of spectra, and as much of this type of work has been done in the past most of the commercial instruments are designed primarily for automatic recording. The opinion appears to be prevalent that a "self-registering microphotometer" is far superior to any instrument which can be read directly. That this is not true for all types of work will appear on consideration of the fact that in many cases flat topped lines can be used, and a single reading will give its total intensity. The recording feature is in fact undesirable for most work on multiplet intensities; this is shown in an amusing manner by the action of one investigator who, in order to avoid the delay of developing, drying, and measuring the record, stuck pins into the scale of the recorder to mark the maximum excursions of the galvanometer beam. Direct reading with the telescope has the advantage that (1) parts of the spectrum which need not be measured can be quickly passed over, (2) development and drying of records is avoided, (3) zero drift, if it exist, can be controlled more easily, (4) greater accuracy is obtained, since a 1000 mm scale can be read directly with the telescope (except possibly in the case of string electrometers), whereas the record is generally only 120 to 200 mm wide, and must be measured later in any case with a traveling microscope or comparator. We are of course assuming that the deflecting instrument is sufficiently stable to make this accuracy worth while. Often over half the time of measurement of a plate can be saved where direct reading is used. For this reason those contemplating line intensity determinations should arrange, where possible, to have their microphotometers equipped to read deflections directly, in addition to the recording feature.

The outstanding commercial instruments are the photoelectric microphotometers made by Krüss (Koch-Goos type) and by Zeiss (modified Koch), and the thermoelectric instrument (Moll) made by Kipp and Zonen, which has recently been greatly improved.

All microphotometers of the ordinary type possess a number of common features. Light from a constant source, almost always an incandescent lamp, is focused by a lens on a slit. In certain cases this slit is almost in contact with the emulsion being measured; in others, an image of the slit is thrown by a second lens on the emulsion. (Sometimes an image of the lamp filament is thrown directly on the emulsion, but this greatly reduces the resolving power of the instrument, and may cause errors due to scattered light.) After passing through the emulsion the light is focused by another lens on a device in which radiant energy produces or controls electrical energy, generally a thermocouple or photoelectric cell. This is connected to and produces deflections in a galvanometer or electrometer.

The parts needed are: (1) A steady source of moderate intensity emitting any wave lengths to which the sensitive instrument responds, (2) A means of passing a beam of light from this through a small area of the plate, generally about 3 mm by 0.1 mm or less, (3) a light sensitive instrument of uniform and quick response which is unaffected by external disturbances, (4) a sensitive current measuring device of short period, (5) a method of holding the plate or film and moving it across the light beam, with a means of observing the plate while this is being done, if possible, (6) a shutter to turn the light beam on and off.

A close race for popularity has been run in the past fifteen years between thermoelectric and photoelectric microphotometers. For some time the thermoelectric instruments were in more common use, but the improvement of the photoelectric cell and of vacuum tube amplifiers has caused a growing trend toward the other type. Its advantages are higher sensitivity, quicker response, and greater ease of insulation from external disturbances; its disadvantages have previously been instability and fatigue, which are now being overcome.

b. *Thermoelectric Microphotometers.* I have discussed the microphotometer question somewhat in a previous paper,[47] although modern developments have been so rapid that this is out of date. The Moll microphotometer is the best known type, and for many years was the only one manufactured commercially. Its popularity is largely due to the great sensitivity and quick action of the Moll vacuum thermo-

element and to the short period and stability of the Moll galvanometer. It is somewhat inconvenient to use for line intensity work since the plate is in a difficult position to observe while running through the spectrum. This general objection applies to any instrument with a horizontal optical system except those of the projection type described below. Other thermoelectric instruments have been described by various writers.[48]

Where an image of the illuminated slit is thrown on the plate care must be taken that the lens used is achromatic in both the visible and the short infrared, as the main portion of the energy used is in the range 0.7–1.5μ, and if all of the light is not in accurate focus the resolving power will be cut down unless a contact slit is used.[47] This is one advantage of the photoelectric instrument, which generally responds to a narrow band of light in the blue and violet regions only. The density values given by the two types of instrument for a given line will of course be very different, since the transmission and scattering of light by silver changes greatly with wave length. For this reason any given plate should be completely measured with a single instrument. Changing the lamp brightness to vary the sensitivity of the instrument by more than a small amount is generally not legitimate with the thermoelectric apparatus, due to the change in effective color of the light beam with temperature.

Vacuum thermocouples can be obtained commercially which react in less than a second, and bismuth-silver or other types of thermocouples or thermopiles which require a few seconds to give maximum response can be built in the laboratory.[1,49] They are used in conjunction with a low resistance high sensitivity galvanometer, of period preferably less than six seconds. For direct reading this length of time is generally needed to reset on another line and to record data; for self-registering instruments it is somewhat long, as an hour or more would be required to trace the record of a 10 inch plate, too great speed causing errors due to galvanometer lag. The newest Moll instruments have been speeded up greatly in this respect, but they are still somewhat slower than the photoelectric cell instruments using a string electrometer, where the period is extremely short and a whole plate can be recorded in from five to ten minutes.

A radiomicrometer can be constructed in the laboratory which will serve as a very sensitive recording instrument, obviating the use of a thermocouple and galvanometer. A densitometer has been described

utilizing this device.[50] It is to be recommended only where economy of cost is the chief object, as the optical system required, though simple, is cumbersome to handle.

A combination comparator-microphotometer is often useful, and one which has given good results for some time in the Stanford laboratory has been described recently.[51]

c. *Photolectric Microphotometers.* The photoelectric cell is much more sensitive and quicker in response than the thermopile or the vacuum thermoelement, especially when used with an electrometer of short period. The string electrometer lends itself admirably to recording work, since its period is extremely short, but it is rather unsatisfactory for direct reading since the scale is so limited. Linear scale deflections are not required in any microphotometer, since absolute densities are not being determined. What is wanted is some single-valued function of the density and intensity which can be conveniently used for interpolation. The quadrant electrometer as ordinarily used has a rather long period, and it is somewhat more satisfactory to use a high resistance galvanometer of short period, with a vacuum tube amplifier if necessary.

Photoelectric cells of the gas-filled type are from ten to twenty times as sensitive as the vacuum type, but are subject to fatigue when run at voltages anywhere in the neighborhood of the flash point. This fatigue can usually be greatly decreased by running the cell on a very low voltage, with a resulting decrease of sensitivity, of course.

The various precautions which must be used in the handling of such cells, and the electrical circuits used with them, can be obtained from the references to articles dealing with photoelectric microphotometers.[52] The potassium hydride cell is in most common use for this purpose, although some of the new dielectric-treated cells[53] may be considerably better in certain respects.

In the original Koch microphotometer, which was the progenitor of practically all of the modern types, the photoelectric cell was used directly as the thermocouple is now, with of course proper differences in circuit. Koch[53a] introduced an ingenious method of compensating for lamp variations by using a second photoelectric cell as a leak across the first instead of the usual fixed leak. This was fed from the microphotometer lamp, and flucatuations in the latter were largely nullified. Whether compensation is used or not a cell makes a good and simple variable leak.

Baker[54] has described a method of using the photoelectric cell in such a way that the effects of variations in the voltage applied to the cell are eliminated. Dobson[55] designed a null photoelectric instrument eliminating errors due to both battery voltage and lamp variations, in which all densities could be measured with equal accuracy. Such null instruments are extremely precise, but there are disadvantages in having to make a balance and then read the setting of the instrument as well.

It appears that a most desirable microphotometer could be constructed on the principle of the projection lantern. A large image of the plate being measured could be thrown on a screen in the usual way, and a small portion of the light in the projected beam could be allowed to penetrate the screen through an adjustable slit, and made to fall on a sensitive instrument. The thermoelement is not sensitive enough for this purpose, and it was not until the newer photoelectric cell amplifier circuits were developed that it was thought advisable to try such a method. Recently Hesthal and the writer[56] have constructed such a device, and in its preliminary stages it has fulfilled all expectations. The projection lamp is run from a storage battery for constancy. The circuit described by Wynn-Williams and others[57] consisting of a balanced bridge containing two vacuum tubes gives about 5000-fold amplification, so that the two photoelectric cells connected in the Koch manner can be run on 6 volts, which gives ample sensitivity and makes fatigue negligible. The slit in the screen is about 5 cm long and is adjustable in width, giving extremely high or low resolving power as desired. The light passing through the slit strikes a lens 6 cm in diameter, and is brought to a partial focus on the main photoelectric cell.

A high resistance galvanometer of moderate sensitivity and short period is used, and its deflections are read by the method described by Smith.[58] This optical arrangement is far superior to the usual ground-glass scale, lamp-filament image method, and is much easier on the eyes than the telescope and scale. Modifications are now in progress designed to make the instrument read intensities instead of densities directly from a given plate, and a complete description will be postponed until these are perfected.

In utilizing the projection principle this instrument is somewhat like that of Koch and Goos,[52] manufactured by Krüss. That, however, being portable, is necessarily on a small scale, and there are numerous advantages in using large lenses of long focus in such an apparatus, as small variations in the thickness of the plate, etc. make less difference.

The advantage of being able to sit in a comfortable position and see on the screen a large area of the plate around that portion being measured, while an easily followed index moves quickly to a number representing the density of the line being measured, can be appreciated by those having experience with the tedious work of densitometering. Small irregularities in the plate or emulsion, which are represented as spectral lines by the registering type of microphotometer, are made evident to the eye, and identification of real spectral lines is greatly facilitated. A wave-length scale printed on rubber can be stretched along the screen when desired, and adjusted for dispersion variations.

d. *Other Types of Microphotometer.* The selenium and thalofide cells have occasionally been used in microphotometers.[59] These, especially the former, show a considerable lag in response, and their sensitivity depends on their previous history. They appear to possess no advantages for this particular type of work not possessed by the thermoelement or the photoelectric cell.

Abe[66] has devised a wedge microphotometer, in which the spectrum to be measured is placed over a slit through which a beam of light passes, and an optical wedge is placed between it and another photographic plate. Upper plate and wedge are moved slowly along by a motor, each wave length on the lower spectrogram being photographed on the upper plate through various parts of the wedge. The record is then cut with ferrocyanide and a definite outline is obtained representing the density distribution in the original plate. It is of course necessary to use one of the ordinary methods to reduce these densities to intensities.

e. *The Microphotometry of Spectrum Plates.* Each reading of the microphotometer consists of a determination of the ratio of the deflection through the clear plate to that through a dense portion. Ideally any portion of the clear plate would give the same deflection when placed over the slit; actually a variation is usually found due to changes in fog, in the emulsion, and in the glass or celluloid backing the emulsion. Eberhard effect may also produce a variation in the clear readings. The approximation which seems to give the best practical results is to choose one particular spot on the plate which seems quite uniform and clear as the standard position for making the "clear reading," and to return always to it. The deflection of the instrument obtained through this clear spot will be referred to as the "100 reading," for the reason that the work of reduction is greatly simplified if the microphotometer

light is adjusted in intensity to give a deflection of exactly 100 cm here. Variations in lamp brightness can then be observed and compensated for by repeated returns to this spot. With a good instrument the 100 reading need not be checked oftener than every five or ten minutes after a steady state has been reached. The rheostat used to control the lamp brightness should have very positive contacts. An easily controlled rheostat is made by stretching a single Nichrome wire along one side of the control table and providing a clamp contact to slide along it. All connections to storage batteries, rheostat, and lamp should be soldered where possible.

The zero deflection of the instrument should be frequently tested to compensate for drift. With most instruments ten or fifteen minutes of use is required before a steady state is reached. The zero need be tested no oftener than the 100 deflection after this is attained, provided, in the case of thermo-electric outfits, that the circuit is well insulated from air currents.

The shutter for producing the zero should be so placed in the optical system that it will not interfere with the field of view, and will not cause an error by giving a false zero. This generally indicates that it should be placed somewhere in the beam before it strikes the plate, so that stray light which affects the sensitive device during readings will continue to affect it while the zero is being taken. Shutters immediately in front of the photoelectric cell are to be avoided for this reason; this is still more true when thermoelements are being used, since in addition to this effect the shutter may emit radiation which causes a false deflection.

After the 100 deflection has been adjusted, readings are taken through the lines, and if any continuous background exists, on one or both sides of each line. Usually when running through a spectrum the background is found sufficiently uniform so that background readings need only be taken for every ten or fifteen lines. The ratio 100 reading/line reading then gives the opacity of the line, and the common logarithm of this gives the density.

We need not plot $d-\log I$ curves to reduce the data, since any simple function of density and intensity will do. If clear and zero readings are constant, we may plot $\log G$ against $\log I$ of the equivalent calibration exposure, where G is the galvanometer deflection through the calibration line. This gives a curve similar to the characteristic curve, but inverted, as in Fig. 9. By plotting on double log paper the galvanometer de-

flections directly against the transmissions of the devices used to vary the intensity in making the calibration exposures, we can read intensities directly from the curve by interpolating the galvanometer deflections for the unknown lines. This can also be done for the backgrounds, and the background intensities may then be subtracted directly. A similar process is gone through for the standardization exposures, but in addition to getting their intensities J from the plate, on the basis of the calibration curves of that plate, we already know

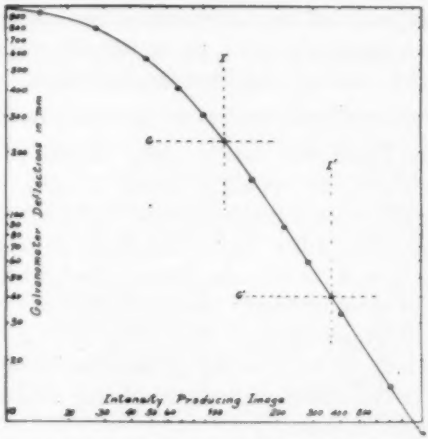

FIG. 9. *Inverted characteristic curve, obtained by plotting the galvanometer deflections of the microphotometer for the various calibration exposures against the transmission factors of the screens or filters in producing these exposures, on double logarithmic paper. In this way the intensities I and I' of the lines producing galvanometer deflections G and G' can be read off directly.*

their true intensities I. A curve relating I/J to wave length for a range of 100 or so Angstroms on each side of each calibration curve is then plotted, and the proper I/J value is interpolated for the wave length of each unknown line from this curve. The J value for each line is then multiplied by I/J, the resulting values of I being on the same basis for all parts of each plate and for all plates which have had the same exposure to the standard source. This process can be repeated for each standardization exposure, but in practice it is more convenient to put all the standardization exposures on the same I basis by means of the calibration curves, and to use the resulting average I values in the reduction.

The actual errors of using the microphotometer are generally less than 2%, most of this being caused by difficulties of setting. Readings on the same setting generally agree to 0.5% with a good instrument. Variation in the emulsion and development are likely to be from 3 to 5% but these quickly compensate one another when a fair number of determinations of a single quantity are averaged. Much larger errors are introduced by the presence of a variable background of intensity greater than 10% of the line intensity.

The procedure outlined above applies equally well to the reduction of photographically recorded microphotometer records, except that here it is less easy to apply the methods of simplification which can be used when the clear and zero readings are controlled.

IV. SPECIAL METHODS OF PHOTOGRAPHIC PHOTOMETRY

1. *Methods for Calibrating with a Single Exposure.* a. *General.* It is obvious that much time would be saved if, instead of taking from four to twenty calibration exposures, these could all be taken at once. This is especially true if the time of exposure is longer than a few minutes; in certain work, as with the Raman effect, where the exposures may be a week or more in length, such a method is a necessity. With a stigmatic spectrograph having a fairly long slit, this can be given a variable width along its length when calibrations are made with a continuous source, the resulting spectrum consisting of several long bands of known difference in intensity.

b. *Step-weakeners.* The same end can be attained somewhat more conveniently by placing in front of a fairly long slit of the usual type a step-weakener, made by building up steps of decreasing transmission by superposing layers of (more or less) neutral material. Plates of quartz coated with antimony or platinum in steps of different densities should be useful throughout the visible and ultraviolet. Or an image of the step-weakener can be thrown on the slit by an extra lens. Neutrality can be obtained by using step-diaphragms ("Rasters") of the types used by Hansen and by Frerichs.[60] With all these methods it is necessary to see that the step-weakener is uniformly illuminated.

c. *The Wedge Method.* The optical wedge may be considered as a step-weakener with an infinite number of steps of infinitesimal width. It can be used wherever the step-weakener is usable, which means in front of the slit in a stigmatic spectrograph, or in contact with the plate in an astigmatic mounting. Its use is discussed in detail by

Dobson, Griffith, and Harrison[6] and the method has been used by a number of workers.[60a] The platinum wedge of Merton is very useful in the ultraviolet.[61] The wedge is slightly less convenient to use than the step-weakener, since the position of measurement along each spectral line must be found and no uniform areas of density exist, whereas with the latter each step is definitely marked. The wedge is extremely useful for showing qualitatively the spectral energy sensitivity of photographic materials; as an accurate quantitative tool it is probably slightly inferior in accuracy to a number of other methods. In the visible region wedges can be obtained commercially made from approximately neutral glass or dyed gelatine. They are generally most easily calibrated in the position of use by means of neutral screens.

d. *Intensity Distribution from Coarse Gratings.* Parkhurst and Farnsworth[62] have made use of the theoretical intensity distribution of the light from coarse gratings among various orders. The grating spectrum is produced at right angles to the principal spectrum, each line being broken up into a number of small spots of known relative intensity.

e. *Line Intensity Distribution.* The theoretical intensity distribution in a spectral line as produced by a given spectrograph gives a calibration with a single exposure, as explained by Ornstein and Minnaert.[63]

Besides the saving of time, all of these methods have the advantage that the calibration spectrum and that being measured are automatically made to have the proper relative density ranges, the required wave lengths are always present in the calibrating source, and it presents the same intermittency characteristics as the source being measured. These calibration methods would undoubtedly displace those requiring successive calibration exposures, were it not for the fact that they present certain optical difficulties in many cases. Where they can be used they are most convenient.

2. Limiting Density Methods. a. *The Outline Wedge Method.* Merton and Nicholson[64] describe a method of using the optical wedge which avoids the use of a microphotometer. There is always some more or less well defined intensity which will give a minimum visible density; by the use of a wedge a large number of densities are produced and the one of vanishing visibility is taken as a standard. Knowing the intensity distribution produced by the wedge, if the length of the spectral line from its base to its last point of visible density be measured, one can obtain the intensity of the line. The method is not of the high-

est accuracy, since in practice the density of the lowest visibility is not sharply marked. To make the line of demarcation more definite recourse is had to copying on a more contrasty emulsion, and to cutting with reducers. This has the disadvantage of all copying processes, that each time an extra photographic emulsion is involved in the reduction new errors are introduced due to the variation of the emulsion. For rapid semi-quantitative determinations of spectral intensities the method is very useful.

b. *Merton's Grating Method.*[65] By interposing a coarse grating in the spectrum in connection with an optical wedge, each line is broken up into a number of small dots which gradually fade out. By counting the number of visible dots a rough value of the intensity of the line is arrived at.

3. *Methods primarily for Use in Absorption or Reflection Measurements (Special Homochromatic Methods). a. The Hilger Sector Photometer*[67] Although the use of the rotating sector disc introduces both intermittency and reciprocity errors,[68] under certain conditions these can apparently be made negligible.[69] Use is made of this fact in the apparatus manufactured by Hilger, designed for measurements of absorption in the visible and ultraviolet by a method similar to that of Henri. Light from a source, which need not be constant, is divided into two beams by a bi-prism, one beam passing through the absorbing material and a rotating sector of fixed aperture, and the other through a rotating sector of variable aperture. The two beams are then sent into a spectrograph in such a way that they produce contiguous spectra. A series of spectrograms is taken for various settings of the variable sector, and points where the densities are equal in the two are found. The amount of light transmitted at each wave length by the material under study can then be found from the setting of the variable sector producing the same density. Both spectra are produced by intermittent exposures, but there is a difference in the ratio of exposure to rest period in the two cases.[70] However, under the conditions of ordinary use the method gives results which are in good agreement with those obtained by other methods.

b. *Lewis' Sector Photometer.*[71] Lewis has designed a modification of the Hilger method which avoids the use of intermittent exposures and causes the radiant energy to pass through the absorptive material in a parallel beam.

c. *Jones' Method.* L. A. Jones[72] has devised an ingenious method for rapidly determining absorption curves of materials. Alternate strips

of the photographic plate exposed in the spectrograph are covered with a grid. Each strip is exposed to a portion of a continuous spectrum whose intensity is increased logarithmically from top to bottom by moving the plate up and down while a radial sector diaphragm is being opened and closed on the collimating lens of the spectrograph. The grid is then moved so as to cover the exposed strips of the plate and uncover the unexposed, and the plate is exposed to the spectrum taken through the absorptive material being studied, while the intensity is varied logarithmically in the opposite direction to the previous one. A series of narrow bands having a graduated density in one direction interlaced with another series having a graduated density in the opposite direction is obtained, and it is a simple matter to match points of equal density very accurately. These points, when joined by a continuous curve, give the absorption curve of the material in the spectral region under observation.

d. *Other Methods.* While numerous other methods of photographic photometry have been proposed, most of them vary only slightly in principle from those described, and reference should be made to the original articles. A number of general discussions of the problems involved and methods of their solution are included in the references, many of these dealing with both heterochromatic and homochromatic photometry.[73]

V. SPECIAL SPECTRAL REGIONS

The problems encountered in special spectral regions are usually those of photography rather than photometry, since the methods described can generally be used at any wave lengths provided suitable sources and other equipment are provided. Certain methods lend themselves better mechanically to one region than another, however.

1. *The Infrared.* Photographs on plates stained with dyes[74] have been obtained up to 12,000A and beyond, and photometry could undoubtedly be done in this region by the photographic method. The methods for taking a single calibration exposure should be useful here, as the exposure times required are very long. Schoen[75] has extended a homochromatic method into the infrared as far as 9000A, and there seems to be no natural limit at this point with the newer dyes. The method could also doubtless be extended to include heterochromatic photometry, the chief problem being the intensity of the light sources.

2. *The Visible.* Glass apparatus and the ordinary dry plate can be used in the region 5200–3500, the glass providing the lower but not the

upper limit. Panchromatic plates are necessary for extending spectra into the red as far as 7200A; orthochromatic plates reach 6000A, and special Astronomical Green Sensitive Plates can be obtained for increased sensitivity around 5400A. The subjective methods described can of course be supplemented by eye methods in this region alone.

3. *The Ultraviolet.* Certain glasses can now be obtained which are transparent down to 2500A,[76] and fused and crystal quartz can be used from far in the infrared down to about 2000A, the crystal quartz being somewhat more transparent at the shorter wave lengths than the other. Rock-salt equipment can also be used in the visible, infrared, and ultraviolet. Water prisms and lenses can be used as far down as 1800A when enclosed in thin quartz walls. Fluorite can be used from the infrared down to about 1200A, and is often useful for thin windows and for correcting lenses. Clear thick pieces of this material are rather expensive.

Fewer filters are available for the ultraviolet region than for the others, although many solutions[77] in quartz vessels can be used. The sputtering or evaporating of metallic coats on quartz or fluorite for this purpose needs much investigation.

4. *The Schumann Region.* Below 2000A the absorption of the atmosphere is very great, so vacuum spectrographs must be used. Practically no quantitative intensity measurements on individual line intensities have been made in the region below 2000A, a small amount of reflection and absorption work having been done.[78] While general photometry would be difficult, it is by no means impossible. The new vacuum spectrographs of higher dispersion now available[79] in a number of laboratories should be of great value in this regard.

The ordinary photographic emulsion is not sensitive below about 1800A due to the absorption of the gelatine. The sensitivity begins to decrease rapidly below 2500A. Schumann plates[80] which contain relatively little gelatine can be obtained commerically or they can be made in the laboratory.[81] They are generally not suitable for intensity work as the emulsion is apt to be very irregular in sensitivity. Duclaux and Jeantet[82] have dissolved the gelatine from ordinary emulsions with acid, producing sensitive plates; the writer has seen no data on their uniformity. Plates coated with fluorescent oils have been investigated for uniformity[83,78] and found satisfactory for intensity work. The writer understands, however, that the Eastman Kodak Laboratories have succeeded in dyeing emulsions so that they are sensitive in the

extreme ultraviolet, and these will doubtless be found superior to any others for intensity work in this region.

The principal problems in the region of short wave lengths would seem to be in the accurate determination of calibration and standardization curves. Work is now in progress on this in the Stanford laboratory.

5. *The X-ray Region.* Photographic photometry can be satisfactorily carried on in the x-ray region, although standardization of plates is rather difficult, both due to the scarcity of accurate means of measuring energy distribution with wave length, and because the sensitivity of the emulsion has abrupt discontinuities at certain wave lengths. The reciprocity law seems to hold quite accurately under many conditions[85] and makes calibration easy, since it can be tested in any given case by comparing the densities obtained with lines of known relative intensity, such as the $K\alpha_1$ and α_2 lines. Here self-reversal need not be feared, and the relative intensities of numerous lines in the x-ray region have been measured for a number of materials.[86] Care should of course be taken not to choose lines which vary in relative intensity with excitation conditions.

Step filters for use in this region can be readily constructed from thin sheets of aluminium or other material, these being calibrated by means of an ionization chamber. Other methods will readily be suggested by a consideration of those used in the visible region.

The characteristic curves obtained with x-rays are of the same general type as those obtained with light, but equivalent phenomana take place at higher densities in the former case.[87] The under-exposure region frequently extends up to density two or three, and it is this region which is most frequently used and is most convenient for photometry. Density is practically proportional to intensity I instead of to $\log I$ in the toe of the curve, so it is more useful to plot d-I curves than the usual d-$\log I$. The departure from this law at higher densities is gradual, however, and care must be taken to accurately follow the true course of the density-intensity relation.

A good general discussion of photographic photometry in the x-ray region will be found in Dorgelo's review article.[88]

VI. CONCLUSION

Photographic photometry, when properly carried out, furnishes a very convenient method of measuring spectral light intensities, and data can be obtained with it which at present can be obtained in no

other way. Much work needs to be done in its development, especially in the selection and calibration of sources of standard intensity distribution. It has been developing very rapidly in the past few years, and a vast amount of data on radiation and atomic structure can be obtained by its use. It furnishes one of the most fertile and rapidly growing fields of investigation of the present time, and it is to be hoped that an increasing number of laboratories will see their way open to embarking on major programs of intensity work in the near future.

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A DYNAMIC ATOM MODEL OF THE FIRST ELEVEN ELEMENTS OF THE PERIODIC TABLE

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ABSTRACT

A dynamic model of a Bohr atom is described; the model being capable of representing all of the normal and many of the excited configurations of the first eleven elements of the periodic table. The nucleus and all of the electrons are suspended in space on jets of air, and are moving; some in circles and some in paths resembling ellipses. About fourteen important concepts of the Bohr atom are brought out by the model, among them being the electron grouping, the progression of perihelion, the spinning electron and the quantization of the spin axis in space.

While preparing the "ball on jet" experiment for a demonstration lecture for sophomores it occurred to the writer that light balls might be moved about in space on jets of air in such a manner as to represent the motions of electrons in atoms.

At first a simple model for representing the motions of electrons in helium was made and since it operated satisfactorily and seemed to have a certain value in exciting the interest of students, a more

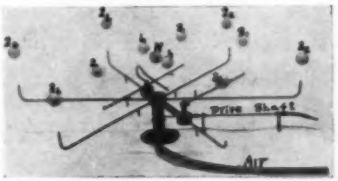

Fig. 1. *Dynamic atom model in operation.*

elaborate model was designed. This model is capable of representing all of the normal and several of the excited configurations of the first eleven atoms of the periodic table.

The design is very simple and a satisfactory idea of the construction may be obtained from a study of the photograph, (Fig. 1.).

DESCRIPTION

The large central vertical shaft carries six radial arms of equal length. These arms are made of one quarter inch brass tubing and are fitted with small nozzles from which issue jets of air. Attached rigidly to the large vertical shaft is a horizontal tube, three fourths of an inch in diameter, with small gear boxes at its ends. This tube

is nine inches long, three inches being on one side of the center of the main shaft and six on the other side. The three inch portion supports a gear box which drives the arm which carries the "elliptic" (2_1) electrons. The six inch portion supports a similar gear box which drives the arm carrying the one "elliptic" (3_1) electron.

The two nozzles above which the (2_1) electrons may be seen, move with unifrom circular motion about their own axis which is itself moving with uniform circular motion about the central axis. The same type of motion is executed by the (3_1) electron.

Immediately above the main vertical axis is suspended the nucleus, marked "N", and above the same horizontal arm are suspended the two (1_1) electrons of the K shell; then come the two (2_1) "elliptical" orbits of the L shell; then the six (2_2) circular orbits of the L shell, completing the closed electron configuration for the neon atom; and last is the (3_1) "elliptical" orbit of the M shell which extends beyond the (2_2) circle at aphelion and in close to the nucleus at perihelion, this being the valence electron of sodium.

Compressed air is furnished through the hose marked "Air" and the rotation is produced by the drive shaft which is attached to a small motor.

The apparatus is mounted on a sheet of varnished plyboard 36″ X 36″ for protection and support.

A very interesting feature of the model is the manner in which the electrons are made to "spin." Some time was spent in trying to make the balls spin about a vertical axis but without success. While setting up the model for a demonstration lecture, however, it was thought advisable to incline the plane of rotation so that the students could get a better view of the orbital motions. This was done, and it was observed that the electrons began to spin; about an axis however, parallel to the plane of the board. The spin is caused, of course, by the fact that the ball hangs to the lower side of the jet and the air passes to one side of it more than the other. This, of course, causes the spin vector (s) to be directed perpendicular to the orbital vector (l) and not parallel nor anti-parallel to it, which would be more desirable.

The tilting of the board, however, and the resulting spin of the electrons brought out another interesting thing. The balls used are made of light celluloid and there is a band around the center where the two hemispheres are joined together. This band causes the ball to have its greatest moment of inertia about an axis normal to the great

circle through the band, and when set to spinning the ball will be most stable when spinning about this axis.

The result is that the spin axis seems to take a "preferred orientation in space" and it keeps this same direction of spin as it moves around and around in its orbit. To bring out this odd feature, one half of the ball is colored bright red and the other white. The white half always points toward one side of the lecture room and the red half to the other.

The following fourteen features of atomic structure are brought out by the model:

1. The atomic constituents are moving in space.
2. The nucleus and electrons are very small compared to the size of the atom.
3. The electrons move in circles and "ellipses."
4. The perihelion and aphelion distances are not the same for the different "ellipses."
5. The (3_1) ellipse shows penetration into the L shell.
6. There is a progression of "aphelion" for the "elliptical orbits," which in an atom is caused by the relativity change of mass.
7. The reason for the low ionization of the alkalies is shown.
8. The electrons are spinning.
9. The spin axis keeps a preferred direction in space.
10. All of the normal configurations of the first eleven elements of periodic table may be represented.
11. Many of the excited states of the elements may be shown.
12. The symmetry of the rare gas configurations, helium and neon, is apparent and the reason for their inert character is obvious.
13. The reason for the electro-positive valency of sodium and the electro-negative valency of fluorine is apparent.
14. The idea of "electron grouping" is shown.

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ELECTRIC CURRENT MEASUREMENT IN TERMS OF LENGTH, MASS, AND TIME

AN UNDERGRADUATE EXPERIMENT

By Thomas B. Brown

ABSTRACT

The experiment consists of two parts: In part I the magnetic field produced by electric current flowing in a modified Helmholtz coil is balanced against the field of a permanent bar magnet. In part II this same magnet is swung at the center of the coil as a magnetic pendulum. From these observations, made in terms of length, mass, and time *only*, both the strength of the current and the magnetic moment of the magnet may be computed. Inaccuracy due to an unsteady local "earth's field" is minimized, and a relatively high precision results.

In teaching the fundamental concepts of electro-magnetism it is desirable to employ in the laboratory an experiment, or a sequence of experiments, which will demonstrate in a concrete fashion the manner in which the electrical (electro-magnetic cgs) units are directly derived from the fundamental units of length, mass and time. For this purpose, however, the standard methods employing the current balance are too elaborate and too mathematically involved. To be of value to the undergraduate student, the experiment should not be too difficult, should be based upon familiar and fundamental text-book topics, and should yield results sufficiently precise to command the student's respect and interest.

A combination of the usual magnetometer, magnetic pendulum, and tangent galvanometer experiments serves this purpose fairly well. The first two experiments together measure the value of a magnetic field (the local "earth's field") from which the current constant of the tangent galvanometer may be computed. The tangent galvanometer may then be used to calibrate a D'Arsonval galvanometer or ammeter; or, following the classical method more closely, to determine the electro-chemical equivalent of a metal.

Use of this combination was inaugurated by F. K. Richtmyer at Cornell University a good number of years ago, and has been used here in our laboratory. Here, however, the precision of the results obtained was far from satisfactory, especially since we have to contend with unsteady local fields, and the experiment herein described is the outgrowth of attempts to adapt this original idea to local conditions. Errors due to unsteadiness of the local field are greatly re-

duced by employing the considerably stronger fields produced by a modified Helmholtz double coil, while the D'Arsonval instrument is calibrated simultaneously by using it to measure the current flowing in the double coil. Furthermore, from the combination of these steps and especially from the use of a null method in the magnetometer measurements, there results a further gain in accuracy as well as a gain in ease of procedure.

As a compromise to convenience in construction, the double coil, $H-H$ (Fig. 1), was made square and the coil separation suitably modified. The actual dimensions are 36.0 cm square by 20.5 cm between centers of the coil elements. The axial field is not entirely uniform, the maximum variation for a distance of 5 cm either way from the

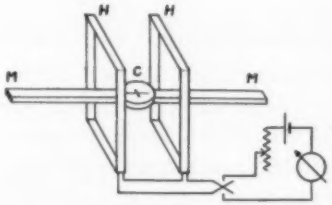

FIG. 1.

center being a little over half of one percent. A circular double coil would be better and will be used in the next apparatus built.

For the first part of the experiment a graduated bar, $M-M$, is placed along the axis of the coil to serve as a magnetometer bar while a sensitive magnetic compass, C, is supported at the center of the coil to indicate the direction of the resultant field at that point. The compass needle is supported on a steel pivot and carries a long aluminum pointer. A silk fibre suspension and a mirror and scale indicator could have been used, and would probably be justified for use with a circular double coil and a steady control field; for the present apparatus the simpler arrangement has proven quite satisfactory. The apparatus is set up with the bar approximately at right angles to the magnetic meridian. The orientation need not be exact, as will be seen from the procedure described below.

The deflection of the needle which results when a current flows through the coil may be exactly neutralized by placing a permanent magnet in the correct position on the magnetometer bar. As in the

analogous magnetometer experiment, for each current value there may be found by trial four such magnet positions—two with the current direct and two with the current reversed. The local field serves merely as a control field so that only its *direction* is involved; and the importance of any unsteadiness of this direction is greatly reduced by the relatively stronger values of the fields being balanced. During these observations the current may be kept constant by observation of the reading of the D'Arsonval instrument. Typical data are presented in table 1.

Since the magnetic field, H, at a point on the axis of a magnet and a distance, D, from the center of the magnet is given by

$$H=\frac{2MD}{(D^2-L^2)^2} \qquad \text{(E. g. Starling, Ch. I.)}$$

and the field, H' produced by the coil, is $H'=GI$, it follows that, when these fields balance,

$$M/I=\frac{G(D^2-L^2)^2}{2D}.$$

$2L$ is the distance between the poles of the magnet, whose magnetic moment is M, while G is the coil constant, derivable from Laplaces law ($dH=\mathrm{I}\cdot ds/r^2\cdot\cos\theta$) and from the geometry of the coil.[1] If as is usually permissible, we assume that the scale, s, of the D'Arsonval instrument is linear, so that $I=ks$, then

$$M/k=\frac{G(D^2-L^2)^2 s}{2D}.$$

The values of M/k for several current values, such as shown in table 1, will of course indicate, by their closeness of agreement, the validity of the assumption of $I=ks$.

For the second part of the experiment the magnetometer bar and the compass are removed from the coil, the coil itself is turned so that its axis is parallel to the magnetic meridian, and the magnet is swung at the center of the coil as a magnetic pendulum. In this

$G=\dfrac{1.6\pi n}{\sqrt{5^3}a}$ for Helmholtz (circular) coils.
a = radius of coil = separation of coils.

$G=\dfrac{0.8\,a^2 n}{(a^2+b^2)\sqrt{2a^2+b^2}}$ for the square coils here used.
$2a$ = side of square.
$2b$ = separation of coils.

n = number of turns (both coils together) for both formulas, and both formulas are for current in amperes.

TABLE 1.

Ammeter reading s	Position of near end end of magnet	Average value of D cm	$M/k=\frac{G(D^2-L^2)^2}{2D}s$
0.750	68.95		
	30.80		
Reversed—	68.80		
	30.85	24.64	912
0.500	—	27.94	904
0.250	—	35.07	914
			Average 910

Compass box is at 50 cm on the bar.
Length of magnet = 11.25 cm. Poles are 0.65 cm and 0.70 cm from their respective ends. $L=4.95$ cm.
For the coil, $a=18.00$ cm, $b=10.25$ cm, and $n=8$ turns. $G=0.1765$.

case it is also desirable to increase the number of turns in the coil both to shorten the period of swing of the pendulum, and more particularly to reduce the importance of the local field, whose *magnitude* now adds to the field of the coil (or subtracts from it, when the current is reversed). In the coil here used, which is designed for current values in the neighborhood of one ampere, eight turns are provided for the first part of the experiment, and forty for the second. If the coil is properly wound this change will alter the value of the coil constant only in the ratio of the number of turns—a factor of 5 in this case.

By swinging the magnet twice for each current value, reversing the current for the second set of observations, the effect of the local field, H_0, may be eliminated. The frequencies for the two cases are given by

$$f^2=\frac{M(G'I+H_0)}{4\pi^2K}, \quad \text{and} \quad f'^2=\frac{M(G'I-H_0)}{4\pi^2K}.$$

whence

$$MI=\frac{2\pi^2K}{G'}(f^2+f'^2).$$

(K = moment of inertia of magnet.)

Again, if the scale of the D'Arsonval instrument is linear,

$$Mk=\frac{2\pi^2K}{G's}(f^2+f'^2).$$

TABLE 2.

Ammeter reading s	Frequency (sec^{-1})		Mk	$\frac{f^2-f'^2}{2}$
	direct (f)	reversed (f')		
0.998	0.2425	0.2065	880	0.0081
0.648	0.2020	0.1580	879	0.0079
0.348	0.1605	0.0986	884	0.0080
00	0.0907	—	—	0.0082
		Average—	881	0.0081

The magnet is 11.25 cm long by 0.6 cm square. Its mass is 36.63 gms and its moment of inertia is 387.0 gm cm^2. For the coil, $n'=40$ turns, and $G'=5\times0.1765=0.882$.

If H_0 may be depended upon to remain fairly constant during the course of all of the observations for this part of the experiment, and if the scale of the D'Arsonval instrument is linear, a more elegant means of averaging these data is to plot a curve, using f^2 values as ordinates and s values as abscissas, plotting reversed values of s as negative, and plotting below the axis (negative values) those values of f'^2 for which the magnet is reversed, with its north pole pointing towards the south. This plot will be a straight line whose slope is $m=MG'k/(4\pi^2K)$, whence

$$Mk \text{ (average value)} = 4\pi^2Km/G'.$$

From the values of M/I and MI for the same value of I, both the magnetic moment, M, and the current, I, may be obtained. Or in general, from the average values of M/k and Mk both the magnetic moment and the instrument constant, k, may be computed. I.e., from the data given in tables 1 and 2

$$M=\sqrt{881\times910}=895 \text{ dyne cm per gauss, and}$$

$$k=\sqrt{881/910}=0.984 \text{ amp per unit scale reading.}$$

A calibration of this instrument using a potentiometer and standard shunt gave $k=0.988$ amperes per unit scale reading. The data given here are taken from a student report and are typical of good student work.

The value of the local field, H_0, may also be obtained as an incidental result. The values of $(f^2-f'^2)/2$ given in the last column of table 2 are proportional to this field and from their average, $H_0=0.138$ gauss.

In performing the experiment the usual precautions are taken to avoid stray fields, and in part two the magnet is swung through a small

angle, not only to insure simple harmonic motion but also to keep it close to the axial field of the coil. The magnets used are about 6 mm square by 8–12 cm long and carefully seasoned. Their poles are fairly well defined and located 5–6 mm from their ends. Other than the non-uniformity of the field of the coil, which has been discussed above, the most probable source of inaccuracy (systematic error) which suggests itself is a possible change of the effective magnetic moment of the magnet as a result of magnetic induction by the field of the coil. Using in place of the magnet a soft iron bar of similar size, an experimental investigation was made from which it appears that this is a negligible source of inaccuracy, at least for the field intensities here used.

As a companion to this experiment, we use one which was also used at Cornell as a companion to the group of experiments discussed at the beginning of this article; namely, the familiar experiment in which the student determines the resistance of an electric heater coil immersed in water contained in a calorimeter. This resistance determination is also in absolute value, although not made directly in terms of fundamental units. Various other experiments, some of them more closely related to the classical methods of standard ohm determination, could also be used as companion experiments.

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A VACUUM TUBE IMPEDANCE BRIDGE

By G. A. Stone

Van der Bijl[1] discusses a circuit which has been modified by A. L. Fitch[2] for use in a vacuum tube potentiometer. The writer has studied this circuit, placing particular emphasis on its incorporation into a convenient piece of laboratory apparatus.

Fig. 1 is a schematic diagram showing the essential features of the impedance bridge. With the switch S closed and no alternating potential on the grids of the tubes, P and P' are adjusted until both the milliammeter A_p and the galvanometer G read zero. This means that

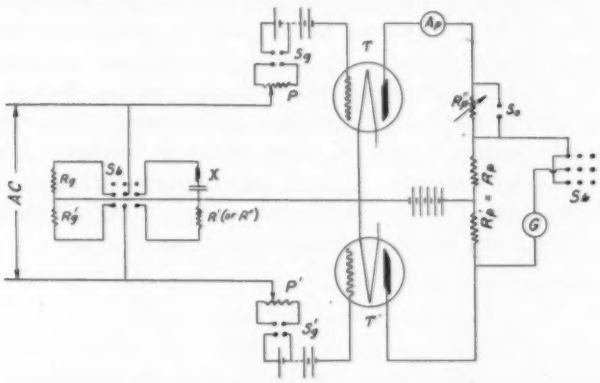

FIG. 1. *Schematic diagram of the impedance bridge.*

both plate currents are zero, and hence the tubes are operating at their cut-off potentials g, as shown in Fig. 4*a*, where the two characteristic curves are superposed with the cut-off potentials coincident. The switch S_b is then thrown upward (see Fig. 3) and an alternating potential impressed across the ends of the equal resistances R_g and R_g' in the grid circuits of the two tubes. Due to this alternating potential impressed on the grids of the tubes, the milliammeter A_p will have a definite reading, and the galvanometer will deflect, since in general the alternating current characteristics[3] of two tubes do not coincide at all points. By

[1] H. J. Van der Bijl. The Thermionic Vacuum Tube. McGraw Hill Book Co. 1920.

[2] A. L. Fitch. A Vacuum Tube Potentiometer. J. O. S. A. & R. S. I., *14*, p. 348; 1927.

[3] The term *alternating current characteristic* is used here to indicate the relation between the effective alternating potential, impressed about the cut-off potential on the grid of the tube, and the effective plate current as read on a direct current measuring instrument placed in the output circuit.

opening the switch S_0 and adjusting R_p'', the characteristic curve of tube T may be depressed until it intersects that of tube T' at the particular alternating potential a (Fig. 4a) which is being applied to the grid of each tube. For this condition, the effective plate currents will be equal, and the galvanometer will read zero.

If the switch S_b is thrown to the lower position, an unknown impedance is placed in the grid circuit of one tube, and the resistance R' (or R'') is placed in the grid circuit of the other tube. The value of the resistance R' (or R'') may be adjusted until the galvanometer reads zero. The value of the unknown impedance is then numerically equal to the pure resistance, *provided that the reading of the milliammeter has not changed.*

This brief description of the impedance bridge and its operation is essentially a description of Mr. Fitch's vacuum tube potentiometer. In his article, Mr. Fitch pointed out the necessity of the milliammeter maintaining a constant reading during the operation of the potentiometer. This condition is rather difficult to attain. In this paper, it will be shown that by slightly altering the procedure, constancy of the milliammeter reading is not necessary for the measurement of impedances.

DESCRIPTION OF APPARATUS

Fig. 2 is a photograph of the impedance bridge. The essentials of the instrument are mounted on a 7″×18″ hard rubber panel in a cabinet

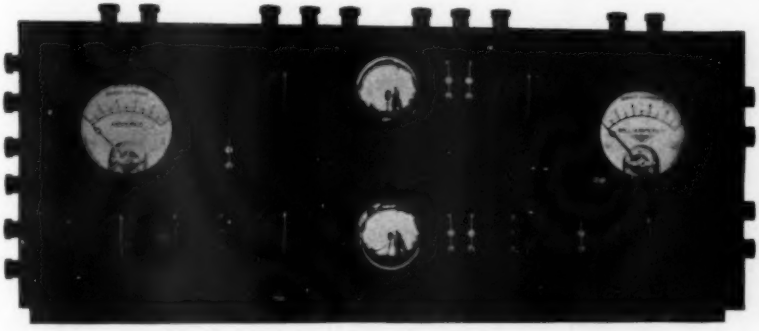

FIG. 2. *Vacuum tube impedance bridge.*

3½″ deep. The binding posts are mounted on the side of the cabinet, and a cover is provided, which can be used without disturbing the external apparatus. The standard laboratory equipment required in ad-

dition to the bridge itself consists of a battery for lighting the filaments of the tubes, a 90 volt plate battery, two 16½ volt grid batteries, a dead beat moving coil galvanometer of about 50 megohms sensitivity, two dial type resistance boxes (0-10000 ohms), and a source of single frequency alternating potential of about 6 volts.

Fig. 3 is a wiring diagram of the internal connections. A 10 ohm rheostat R_f, a switch S_f, and an ammeter A_f (range 0-1.0 amps) are connected in series with the filaments of the two tubes, which are connected in parallel. An adjustable constant potential is applied to the grids of the tubes by means of the batteries G and G' and of two 400 ohm potentiometers P and P'. When the apparatus is not in use, the

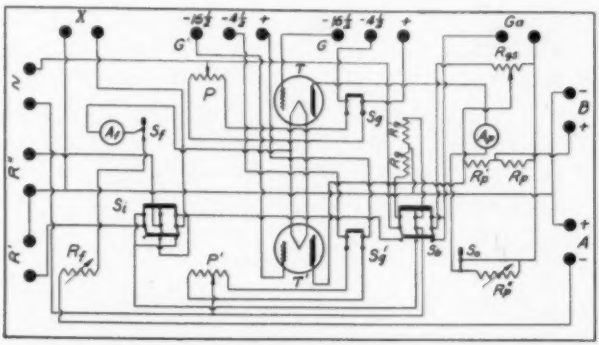

FIG. 3. *Internal connections.*

grid batteries are disconnected by means of the switches S_g and S_g'. The plates of the tubes are connected through two 1000 ohm inductionless coils R_p and R_p' and through a common plate battery to the filaments. A milliammeter A_p (range 0-10 ma) and a compression rheostat R_p'' are connected in the plate circuit of tube T. The switch S_0 serves to reduce the resistance R_p'' rapidly to zero and to return it to its original setting. Across the equal resistances R_p and R_p', a galvanometer is connected through a 5000 ohm potentiometer R_{gs}, used as a universal shunt.

Two 1000 ohm coils R_g and R_g' are connected in the grid circuits in such a manner that when the switch S_b is closed as indicated in Fig. 3, the alternating potential connected at the frequency terminals is impressed across the ends of the coils. In this case, one half of the total drop in potential is impressed on the grid of each tube. The switch S_b is a special "rocker" type switch so designed that the right hand set

of terminals is disconnected before, and connected after the other two sets of terminals. This prevents violent deflections of the galvanometer which would otherwise result when the grid circuits are opened.

When the switch S_b is in the lower position, the alternating potential is applied across the unknown impedance X and one of the two resistance boxes, R' and R''. When the switch S_i, also a "rocker" type switch (not shown in Fig. 1), is closed upward, the drop in potential across X is impressed on the grid of tube T, while the drop in potential across R' is impressed on the grid of tube T'. By throwing this switch to the other position, the unknown X is changed to the grid of T', and the drop in potential across R'' is applied to T.

THEORY AND EXPERIMENTAL RESULTS

It has already been pointed out that the milliammeter A_p must read the same when the resistances R_g and R_g' are connected in the grid circuits as when the unknown impedance and either R' or R'' are connected in these circuits. That is, when the switch S_b is thrown from one position to the other, there must be no change in the alternating potential applied to the grids of the tubes. However, this condition is

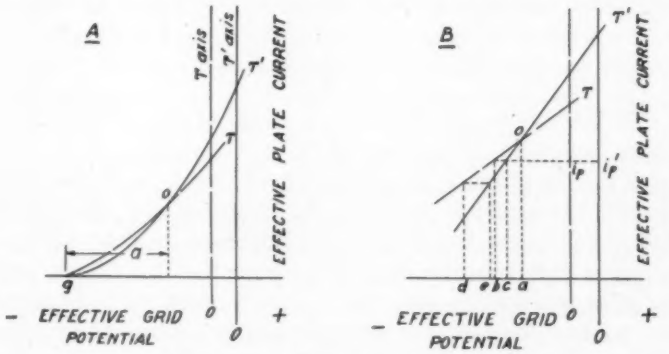

FIG. 4. *Alternating current characteristics of the tubes and output circuits*

very rarely attained, as it is difficult to obtain a simple source of alternating potential which is entirely independent of the load impedance placed across it. From a study of Fig. 4a, it may be seen that for any alternating potential having an effective value other than a applied to the grids of the tubes, the effective plate currents are not the same in the two output circuits. In order to reduce the galvanometer deflection to zero in this case, different alternating potentials must be ap-

plied to the grids of the two tubes, and the reading of R' gives a false value for the unknown impedance.

This apparent difficulty can be obviated in one of two ways. First, by means of a potentiometer, the applied alternating potential can be adjusted for each reading to its original value as indicated by the milliammeter A_p. However, this operation adds to the difficulty of making measurements, and is neither desirable nor necessary. What seems to be a better procedure—one in which the applied alternating potential need not be constant for different loads—is suggested by a further consideration of Fig. 4a and Fig. 4b, which is an enlarged sketch of the small portion of the characteristic curves near the operating point o.

For the definite effective potential a applied to the grid of each tube, the effective plate currents are exactly equal, and the galvanometer does not deflect. The operating point is then located at o. Now suppose the effective alternating potential b is applied to the grid of tube T; and at the same time the alternating potential c is impressed on the grid of tube T'. The effective plate currents in the output circuits of the two tubes will then be given by $i_p = i_p'$, and the galvanometer will read zero. Since for a given current, the drop in potential across an impedance is directly proportional to the impedance, it follows that:

$$\frac{c}{b} = \frac{R'}{X} \tag{1}$$

where R' is the resistance in the input circuit of tube T', and X is the unknown impedance in the input of T.

Let X now be placed in the grid circuit of tube T', and the resistance R'' be substituted for it in the input of T. A setting of R'' different from that of R' will be required to bring the galvanometer deflection to zero. This means that two new alternating potentials have been applied to the grids of the tubes, since the current through the resistance and impedance has changed. Let these new potentials be d and e. Analogous to equation (1), the relation for this new condition may be written:

$$\frac{d}{e} = \frac{R''}{X} \tag{2}$$

X is now the unknown impedance in the grid circuit of tube T', while R'' is the grid leak of tube T.

Multiplication of equation (1) by (2) gives:

$$\frac{c}{b} \cdot \frac{d}{e} = \frac{R'R''}{X^2}. \tag{3}$$

It can be shown by making a suitable approximation that the left hand member of equation (3) reduces to unity when the operating conditions are properly chosen. For this case, the expression for the unknown impedance becomes:

$$X = \sqrt{R' R''} \tag{4}$$

The operation indicated by this discussion may be accomplished by means of the switch S_4. The value of R' is obtained in the usual manner. The switch S_4 is then thrown to the lower position and the value of R'' obtained in an exactly similar manner. In order to prevent the galvanometer from undergoing violent deflection while this operation is being carried out, the switch S_5 should always be closed upward before throwing S_4.

In the derivation of equation (4), certain operating conditions are assumed. First, the deviations of the potentials b, c, d, and e from the value a must be small enough that the curves are essentially linear over this range about o. The second restriction on equation (4) is that the point o must be so located that the angle between the characteristic curves is small. That is, the difference between the slopes of the two curves at their point of intersection must be negligible in comparison with either one of them. (The location of the point o at the potential a is merely for the sake of convenience. It is apparent that any other point which satisfies both of the conditions mentioned will serve equally as well.)

In order to satisfy the first condition when the 60 cycle lighting line is used as a source of potential, it is necessary to exercise some care in choosing a transformer for use with the impedance bridge. Two types of transformer were found successful—one having a rating of 40 watts and an output potential of 6 volts; the other having the same output potential but a rating of 75 watts. Both of these transformers furnished a potential which did not vary more than a few hundredths of a volt for impedances which were placed across their output.

The more nearly matched are the characteristics of the two circuits, the more nearly the second condition will be fulfilled. An optimum value of the angle between the two curves may also be secured by a proper selection of the potential a. If two tubes of the same type are found to have widely different characteristics, it becomes necessary to replace one of them with a tube which more nearly matches the other. Several different tubes of type 201A were used in this work, and the results

were found to agree within a fraction of one per cent. An experimental determination of the alternating current characteristics of the tubes and their output circuits showed agreement with the assumptions which have been made.

In the following table, the three groups of data were selected as typical of the measurements which may be made with the impedance bridge.

TABLE 1. *Representative data taken with impedance bridge.*

Unknown impedance		R'	R''	$X=\sqrt{R'R''}$
Resistance (ohms)	9000	8940	9060	9000
	7000	6960	7040	7000
	5000	4978	5021	5000
	3000	2993	3008	3000
	1000	1000	1000	1000
	500	500	500	500
	50	50	50	50
Inductance (henries)	.005	9.0*	9.0*	9.0
	.010	9.5	9.5	9.5
	.015	10.4	10.5	10.4
	.020	11.5	11.7	11.6
	.025	12.8	13.0	12.9
	.030	14.1	14.5	14.3
	.035	15.6	16.1	15.8
	.040	17.1	17.8	17.4
Capacitance (mfd)	2.0	1254	1397	1323
	1.3	1938	2145	2035
	1.0	2523	2784	2657
	.9	2805	3090	2943
	.7	3628	3950	3783
	.5	5115	5485	5300
	.4	6459	6830	6650
	.3	8710	9005	8860

* These values were obtained from the galvanometer deflections by interpolation.

The curve in Fig. 5 is obtained from the data given in the second group of the table. The ordinates are the squares of the measured impedances, and the abscissas are the squares of the inductance L. These measurements were taken with a Leeds and Northrup inductometer having a direct current resistance of 8.9 ohms. The square root of the intercept of the curve, which should give the direct current resistance, is seen to be 8.84 ohms. From the slope of the curve, which

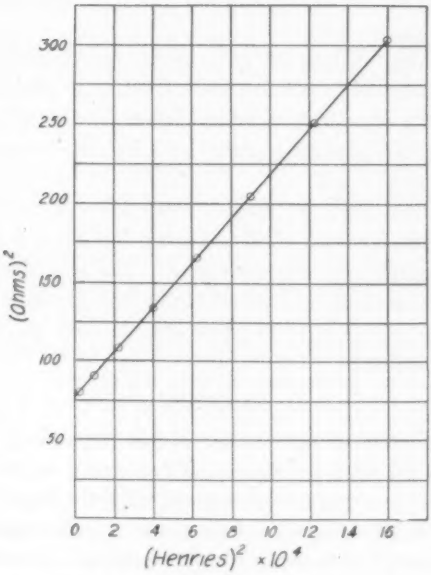

FIG. 5. (Inductometer)

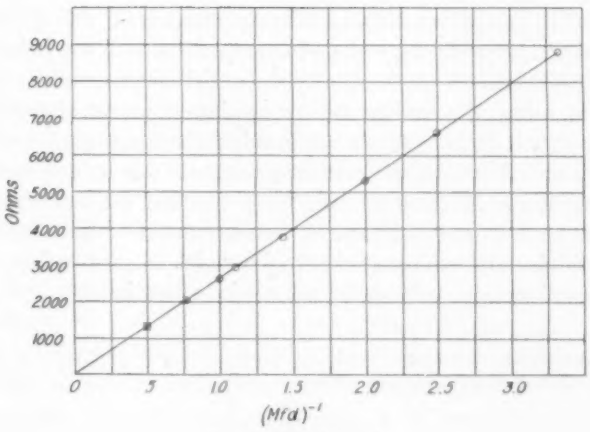

FIG. 6. (Condensers)

should give the square of the angular velocity, the frequency is found to be 59.9 cycles per second.

The third group of data in the table is represented by the curve in Fig. 6. The observed impedances are plotted as ordinates against the reciprocals of the capacitances as abscissas. From the reciprocal of the slope, which should give the angular velocity, the frequency is found to be 60.0 cycles per second.

As a further check on the range of usefulness of the instrument, the impedance of an air core coil of unknown inductance and a condenser of unknown capacitance was measured. The combined impedance of the coil and condenser in series and in parallel was then measured. The observed values were found to agree exactly with those calculated for the two combinations.

SUMMARY

From a number of measurements of the type given in the table, it may be concluded that resistances, inductances, capacitances, or any combination of these may be measured with the impedance bridge to an accuracy of at least one half of one per cent. The smallest impedance which was measured with the bridge has a value of 9.0 ohms. Under this condition, the resistance boxes were carrying an excessive current which produced a slight heating effect. There seems to be no reason why smaller impedances cannot be accurately measured if suitable standard resistances are available. The upper limit of the range may be somewhat extended by using two resistance boxes each for R' and R'' but their value must always be kept small in comparison with the input impedance of the tubes.

Before using the bridge for measuring unknown inductance or capacitance, it is necessary to determine the frequency of the alternating source from the observed impedance of a standard capacitance or inductance. When once this frequency has been determined, it may be used in any subsequent measurements of these quantities.

The advice and encouragement of Dr. M. N. States have largely contributed to this study, and I acknowledge my indebtedness to him.

DEPARTMENT OF PHYSICS
UNIVERSITY OF KENTUCKY
LEXINGTON, KENTUCKY. July, 1929.

THE APPLICATION OF SPARK RECORDING TO EXPERIMENTS IN MECHANICS

By Paul E. Klopsteg

Physical spectroscopy, atomic structure, wave mechanics and the related branches of modern physics have monopolized the interest of so many physicists that unless one is active in these fields he must pause to wonder whether he can qualify as a physicist. Notwithstanding this trend in research, it may be presumed that teaching methods and aids to lecture and laboratory instruction must continue to command some attention in order that the future physicist may become well grounded in the elements and fundamentals of his subject.

If one makes a detailed study of present laboratory methods and apparatus, the conclusion is inevitable that there is room for much improvement. The writer has been fortunate in the opportunity of making such a study, in the course of which an apparatus group in mechanics was developed. This has so simplified the apparatus and the procedure in acceleration experiments, and has so increased the accuracy of the experimental data that a description should prove interesting to those who find the planning and teaching of laboratory methods a satisfying occupation.

The mention of experiments in mechanics brings to mind, among other things, the several devices that have been adapted to the measurement of time, which is an essential factor in most mechanics experiments. The metronome, the pendulum, with or without a telegraph sounder, and the stop-watch and stop-clock have been used. Where a record of displacement with time is needed the most common means is a tuning fork provided with a stylus with which its vibrations can be traced on a smoked surface. To the tuning fork method there are several objections. One of them is the variable friction of the stylus on the recording surface. Another is the uncertainty and variability of the frequency, a value for which the student is required to take on faith because he has insufficient time during the laboratory period to make a separate determination of the frequency. Besides, such a determination would constitute another experiment in the course, requiring special planning and equipment for its accommodation.

In numerous mechanics experiments an exact recording of displacement with time is essential. The ideal method is that of re-

cording successive instantaneous positions by means of equally timed jump sparks, on paper so sensitized that the passage of a spark through it leaves not only a perforation but a prominent mark. There is no dragging stylus to introduce variable friction; there is no tedious counting of undulations in a tuning fork trace; in fact, all the objections to the tuning fork method are absent when spark recording is employed. Its reduction to practice, although beset with the usual difficulties in development, has fulfilled every expectation. As it seems to the writer, it should stimulate new interest in the laboratory study of a most fundamental branch of physics.

Spark recording, although not unknown in its generic aspect, cannot be said to have been suitably developed for everyday laboratory use. It has been employed in experimental ballistics for many years. It has, in a few instances, been used in the laboratory for obtaining the record of motion of a car on an inclined plane,[1] and for obtaining the displacement record of a freely falling ball.[2] That it has not been more widely used may be accounted for by the fact that its possibilities had not been developed.

In 1918 the writer was engaged, with Maj. Alfred L. Loomis, then in charge of the development section at Aberdeen Proving Ground, in the development of a high speed chronograph for the Ordnance Department. In the Aberdeen chronograph, for which that section was responsible, paraffin coated, colored paper strips were used for obtaining velocity records of projectiles. The writer worked out another application of such recording to measuring the time of flight of small arms projectiles. These were the earliest instances of spark recording on paper sensitized as described. Wherever a spark puncture occurs on such paper there appears a relatively large spot that results from the melting or burning of a bit of the paraffin coating by the heat of the spark. Within the area of the spot a minute perforation marks precisely the point at which the spark penetrated the paper.

Apparently the first student laboratory application of spark recording on such coated paper was made by Behr and Reynolds.[3] They developed an apparatus with which a record of displacements of a freely falling body was secured with the aid of a synchronous motor on the shaft of which a commutator was arranged to charge a condenser

[1] Zeleny & Erikson, Manual of Physical Measurements, p. 13; 1923.
[2] Fountain, Laboratory Manual of Practical Physics, exp. 14; 1926.
[3] J.O.S.A. & R.S.I., *13*, p. 213; 1926.

of large capacity and discharge it through a spark coil. The spark-producing circuit was substantially the same as that employed in the Aberdeen chronograph.

At Cornell University where Behr and Reynolds did their work Seemann[4] extended spark recording to a rotational inertia apparatus. In such an apparatus a relatively longer interval is needed than in a free fall. Seemann used the synchronous motor, but geared down the commutator to one revolution per half second. A strip of coated paper on the rim of the rotating disc receives the record.

The group of apparatus for mechanics experiments which resulted from development work begun in our laboratories some time ago has as its nucleus a device for producing equally timed sparks, combined with a device for accurately measuring the interval at any frequency employed. The spark-timing device will be described in detail; the interval-measuring means is the subject of another paper in this Journal.

For producing the equally-timed sparks we adopted a steel bar which is electrically maintained in vibration, making and breaking a battery circuit through the primary of a suitable jump-spark coil. Adjustable masses are employed for varying the frequency of the bar.

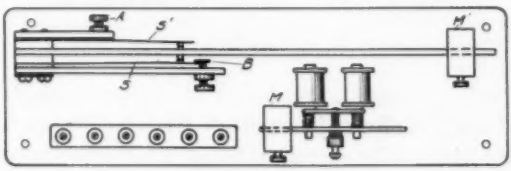

FIG. 1.

Simple as such a device appears to be, many difficulties were encountered before thoroughly reliable operation was secured. As finally designed and constructed, the successive intervals between "breaks" of the contacts through which the spark coil is actuated show a maximum deviation of .04% from the average interval. The investigation by which this constancy of interval was determined was made with an Aberdeen chronograph, with a paper speed of 1250 cm per second.

Fig. 1 is a plan drawing of the spark timer as finally developed and Fig. 2 is a photograph of the apparatus, with an extra bar. Fig. 3

[4] J.O.S.A. & R.S.I., *17*, p. 386; 1928.

is a schematic diagram of the circuit employed in producing sparks at equal intervals.[5] Although the figures are self-explanatory, several important points about the design, which at first were not obvious, should be mentioned.

To secure satisfactory control of amplitude, one may use variable resistance in series with the driving magnet, or the magnet may be mounted so that fine adjustment of the distance between its poles and the bar is secured. The latter method, because of its simplicity, was chosen, and, with the design adopted, leaves nothing to be desired. The "fixed" contact at which the primary circuit of the coil is interrupted is mounted on a relatively stiff spring S, as is also the corresponding contact on the opposite side of the bar, (S') through which the

FIG. 2.

driving magnet is actuated. The latter spring mounting is adjustable, by means of screw A, for adjustment of the contact gap, which determines the duration of the current through the driving magnet. It was found that, to insure equality of the successive time intervals, the coil-actuating contact spring must be damped, and the contact on this spring must not be permitted to follow the bar beyond a certain distance, defined by an adjustable stop, to insure that the break shall occur at the corresponding instant in each successive cycle. The same battery (6-volt storage) is used for driving the bar and for producing the jump sparks. Of five different commercial spark coils and high potential transformers that were tried, only one was found that never failed to produce a spark which would insure a clear record. Binding posts are provided for connection of the battery, spark coil and impulse counter.

[5] In the circuit is included the impulse counter, which is not essential to the operation of the spark timer; it merely counts the vibrations of the bar.

To cover the complete range of short intervals needed in the laboratory, two bars of different cross-section are provided. The interval ranges of these are approximately .037 to .11 second, and .09 to .45 second, respectively. Two masses MM', that can be clamped on either bar, are used for varying the interval within the limits mentioned. The shortest interval obtainable is suitable for experiments with freely falling bodies, whereas the longer intervals are used with apparatus in which acceleration is much less than that produced by gravity in a free fall. The two bars are quickly interchangeable. When the heavier bar is used, the base must be clamped to a stable support.

When the bar vibrates not over 4 or 5 times per second, it is easily possible for the student to count a number of vibrations over a length of time that is measured with a stop-watch. Although higher frequencies can be counted (with practice up to 10 or 11 vps) the student

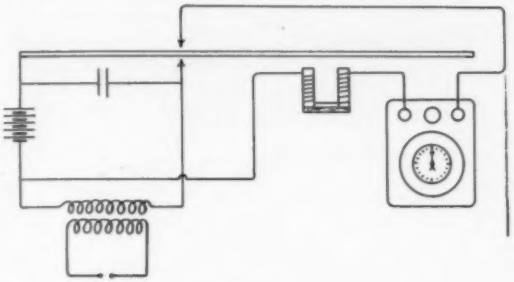

FIG. 3.

could hardly be expected to do this accurately, and he would probably feel uncertain about his results.

The impulse counter enables the student to determine with great accuracy the length of the interval. This he does by connecting the instrument as shown in Fig. 3, and, with his own watch finding the time corresponding to a large number of vibrations registered on the impulse counter. He thus determines for himself with requisite accuracy the most important factor in his calculations—important because the length of the interval enters all acceleration formulas as a squared quantity, so that an error in its value becomes doubled in the final result. Pedagogically any experiment is greatly improved if the student secures all data by individual observation instead of accepting some on faith. This fact greatly enhances the value of the combination of spark timer and impulse counter.

The group of mechanics experiments to which spark recording has been applied is characterized by the fact that the same timing means are employed in every experiment and that in each one the record is made on either a strip or a circular disc of sensitized paper. In the linear acceleration experiments, in which the strip is used, a length of record in excess of a meter is obtainable. This is accomplished in the Behr and Reynolds free fall apparatus by designing the apparatus for a height of fall corresponding to 0.5 second. In other apparatus, such as Atwood's machine, car-and-track, and roller-on-incline, the sensitized paper strip is used as the means through which the force is applied to the masses to be accelerated. The strip takes the place of the string ordinarily used in such apparatus,[6] hence it may be made relatively long. The strip passes over a light pulley, opposite which the spark point is fixed.

FIG. 4.

Fig. 4 shows a car-and-track apparatus, with which accelerated motion may be studied in several ways. A steel bed with two rails, and provided with levelling screws, constitutes the track. The car, with three hardened steel wheels, is fitted with a carrier to hold one, two, three or four 500-gram slotted weights of stock pattern. At one end of the car a clamp is fixed for attaching one end of the paper strip. The free end of the strip carries a similar clamp, to which hook weights may be attached. Both ends of the track are fitted with stationary friction brakes which stop the car without shock or rebound. The pulley end of the track is allowed to project over the edge of the table, so that the weighted end of the strip may pass vertically downward. The spark point, which is adjustable sidewise, is clamped on a bakelite strip. Four records per inch of width may thus be secured on the same record strip,[7] simply by shifting the spark point a quarter inch after each record.

[6] Edwards, H. W., S.S. & M., *23*, p. 536; 1923.

[7] Sensitized paper is obtainable in widths of 1 inch and 2 inches.

With one high tension terminal of the spark coil connected to the grounded track and the other to the spark point, and with the strip properly attached, the spark timer is started and the car released. After the car has travelled the length of the track, a most striking "picture" of the motion is given in the series of marks recorded by the sparks on the strip. The ease and certainty with which such a record is obtainable encourages and stimulates a more detailed and complete experimental study, and promotes more thorough understanding of the force-mass-acceleration relationship than does any other method known to the writer.

The spark point deserves special mention. Owing to occasional "weak" spots in the paper, the spark manifests a tendency to jump elsewhere than to the location on the paper strip directly beneath the spark point. This tendency was corrected by enclosing the spark

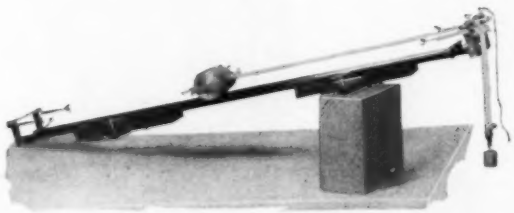

FIG. 5.

point, consisting of a short piece of wire, in a capillary glass tube; the open end of the tube is brought very close to, but not touching, the paper. The spark is constrained by the capillary bore so that it cannot but travel the "straight and narrow" path; its deviations become negligible. The total spark length is about 10 mm. The glass tube permits only about 0.5 mm of the spark to "wander" instead of 10 mm, which would be the case were a metal point brought within 0.5 mm of the paper.

By elevating one end of the track it becomes an excellent inclined "plane," on which either the car of adjustable mass or a massive roller may be used. These experiments give the student practical acquaintance with composition of forces, acceleration, energy relations in motion down the incline, and energy partition between translation and rotation; and he learns methods of allowing for and compensating for friction. The stationary brakes mentioned above serve to catch either the car

or the roller at the end of the run down the incline. Fig. 5 shows the roller-on-incline apparatus.

The Atwood machine which was designed for spark recording is illustrated in Fig. 6. Best results are secured with this apparatus when the paper makes a complete loop, as shown in the illustration. In this manner, especially when the load is small, automatic compensation for change in mass of the paper is secured as the record strip passes over the pulley. One clamp is provided with a metal strip which at the end of the run engages the stationary friction brake

FIG. 6.

shown. The other clamp has a weight carrier. Both clamps may be loaded with hook weights. The accelerating weights, consisting of 5-gram rods, may be removed at any desired point on their downward journey by a vertically adjustable pair of curved arms. A releasing device enables the student to start the motion at the instant a spark occurs, so that velocity and time simultaneously have zero value.

In all these experiments compensation for friction can be made and tested with beautiful simplicity. Weight is applied to the system until uniform motion ensues when an initial velocity is imparted to it. The test for uniformity is made by obtaining a spark record under such conditions, and stepping off the distances between marks with

a pair of dividers. Positive or negative acceleration is instantly revealed, and correction of the adjustment made accordingly.

In Fig. 7 is shown a new form of rotational inertia apparatus in which spark recording is used. A three-step disc has been adopted, to secure three torque-radii. The mounting of the disc brings its axis vertically above the end of the base, to facilitate application of the torque at any radius. Extreme simplification has been made possible by substituting a spark point for the usual tuning fork, and by employing a chart of coated paper on which a scale of single degrees scale is

FIG. 7.

printed. This arrangement[8] does away with the necessity of a divided circle on the disc, with reading magnifiers, and with separate devices for reducing a record to numerical data. The coated paper chart is secured to the inertia disc with a few touches of rubber cement[9] or universal red wax. The accelerating load is applied, the spark timer set going, and the disc released. The spark point carriage is slowly moved on its slide, which moves the point accurately along a radius of the disc

[8] Acknowledgement is made to Professor M. N. States for a helpful suggestion. In his classes he has been using polar coordinate paper, coated with smoke on the reverse side; on this surface a tuning fork trace is made. After fixing, equal groups of undulations in the trace are counted out, and the division points transferred by a needle prick to the coordinate scale on the opposite side of the sheet.

[9] Rubber cement is a most useful laboratory aid.

from rim towards center or vice versa, and the spark record is made along a spiral path. The chart with its record, incorporated in the laboratory report, constitutes the student's evidence for correctness of his numerical data, since every spark perforation is and remains in the degrees scale, from which its angular position is read. The speed with which such records are obtainable permits use of the apparatus by many more students than can be accommodated with apparatus that must remain in use until all records have been reduced to numerical data.

The study and development here described have, in the writer's belief, resulted in a very substantial simplification of laboratory technique in the mechanics experiments described. Adjustments are simple to make; clean records are quickly and easily obtained; the student has the original time-displacement record to embody in his report; the spark timer, coil and impulse counter constitute a timing device applicable to an entire group of mechanics experiments. In addition, the vibrating timing bar in conjunction with the impulse counter, once they have been rated with a watch, constitute a high speed electrical stop-watch useful in any experiment where short intervals must be measured with accuracy greater than that of a stop-watch and stop-clock.

Mr. M. C. Magarian has ably assisted in this work, and much credit is due him for ingenuity in developing many of the details of the apparatus described.

CENTRAL SCIENTIFIC COMPANY,
CHICAGO, ILLINOIS.

AN IMPULSE COUNTER, TALLYING DEVICE AND ELECTRIC "STOPWATCH"—A USEFUL LABORATORY INSTRUMENT

By Paul E. Klopsteg

In another article[1] reference was made to an impulse counter, by means of which it is possible to measure accurately the period of a vibrating steel bar, maintained at constant amplitude by an electromagnet which is periodically energized by the closing of a pair of contacts, one of which is on the bar. The same instrument has been found to have other applications in laboratory procedure, in which it has proved itself most satisfactory.

An electromagnet energized by direct current from a low voltage source operates on a pivoted armature normally held against a stop by spring tension. To the armature is attached a driving pawl which, at each stroke of the armature towards the magnet, advances a 60-tooth ratchet wheel one step. The motion of the ratchet wheel is rendered positive by a check-pawl, and by a device which locks the driving pawl when it is in its advanced position. The mechanism, by virtue of its small moment of inertia and ample actuating power, operates positively and without skips at all frequencies below 125 per second. It is difficult to believe, until one sees it perform, that such a mechanism can follow 125 impulses per second. Light moving parts, properly adjusted, and sufficiently strong impulses, make this surprising performance possible.

The staff of the ratchet wheel carries a "sweep-seconds" hand, to indicate individual impulses on a 60-division dial. Through a train of 30/1 reduction gears it drives an auxiliary hand on a small dial. With this combination the instrument registers individual impulses up to 1800.

To facilitate certain measurements, a mechanical push-button is provided with which the counter can be held inoperative. Releasing the button allows the driving pawl to operate on the ratchet. In this manner the student may, with his own watch, determine accurately the frequency of equally-timed impulses, and hence the time interval in which he is interested. A view of the instrument is shown in Fig. 1, nd, with cover removed, in Fig. 2.

[1] J. O.S.A. & R.S.I., *19*, pp. 335–344; 1929.

Some of the laboratory applications of this useful device may be suggested. Perhaps the simplest is its use as a "remote" tallying or counting device, actuated either automatically or by hand. Automatic operation is possible when recurring electrical impulses of constant or variable interval can be produced by the apparatus from which data are to be obtained. For example, one might wish to know the number of "makes" and "breaks" during a given period of time of some form of electrical control apparatus, actuated by temperature or pressure variations. This can be readily accomplished by connecting the counter in the relay circuit with a suitable source of current

FIG. 1.

FIG. 2.

and noting the dial readings at the beginning and end of the period of observation. In some instances the number of revolutions, either total or in a certain time, of a revolving part, or the number of strokes of reciprocating part, may be required. A contact actuated by the moving part may be used for operation of the counter. The fact that the counter responds to 125 impulses per second, or 7500 per minute, and that it can be located at a distance from the apparatus being investigated, makes it adaptable to some conditions in which an ordinary mechanical counter cannot be used.

For the measurement of vibration frequency of a bar or of a tuning fork, electrically maintained, the impulse counter is especially satisfactory. When applied to the vibrating bar of the spark timer, described in another paper in this Journal[3], the number of impulses in a certain time, as determined by means of a watch, is accurately counted. From

[3] Loc. cit.

the elapsed time and the total number of impulses the student computes the length of interval between successive breaks of the contact on the bar, which is the time interval between successive recording sparks. In the article mentioned the pedagogical importance of having the student obtain all data by direct, personal observation was pointed out.

Melde's experiment assumes new significance if the student can determine for himself the frequency of the fork under conditions of loading, while it is producing standing waves on the string. The impulse counter, suitably connected in the driving circuit of the fork, measures the frequency as before; this constitutes an actual, convincing check on the frequency value computed from the tension and the linear density of the vibrating string. The rating of an electrically driven fork with the impulse counter is a measurement of great accuracy, with the results of which the frequency value determined by the usual laboratory methods can be compared. In fact, for forks below 125 vps, the impulse counter constitutes a method of rating as accurate as the phonic wheel, and simpler in application.

The impulse counter may also be used for measuring the number of alternations per second of alternating current of 60 cycles or less. The secondary of a low-voltage transformer is connected to the counter. When the primary is connected to a 60-cycle supply line, the counter receives 120 impulses per second, which is double the frequency. It counts these indefinitely, without error.

Another method of measuring the frequency of an alternating current with the aid of the impulse counter is indirect, but even more instructive to the student. Its greater value lies in the fact that it gives the student a practical introduction to resonance and to the laws of vibrating strings. Reference to Figs. 3 and 4 will make clear this method.

Fig. 3 is an elevation drawing showing an iron wire W so supported that known tension can be applied to it, as shown. Two bridges define the vibrating section of the wire, of measurable length. At the mid-point between the bridges, a light steel point P is clamped to the wire, its function being that of a circuit closing device. To maintain the wire in vibration, an electromagnetic device L is provided. This consists of a base with support, on which are carried the electromagnet E and mercury cup M, both of which are vertically adjustible by slow-motion screws. These permit great nicety of adjustment to secure the the most suitable amplitude of vibration. As the wire vibrates, contact P momentarily dips into the mercury at each downward motion.

Fig. 4 is a schematic diagram of the circuit used in the experiment of measuring ac frequency. In addition to the parts referred to in connection with Fig. 3, there are shown a stepdown transformer T, the battery B, the rheostat R, and the impulse counter C. The low side of the transformer is connected directly to electromagnet E. Weights are applied to the weight hanger, and the bridges are adjusted until the wire vibrates in resonance with the alternating current which is

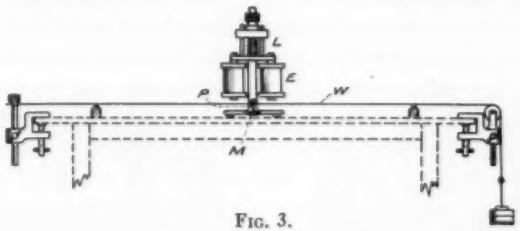

FIG. 3.

energizing the magnet. The counter then registers the number of times the circuit is closed in the mercury cup. On 60-cycle current, it is theoretically possible to obtain 120, 60, 30 or 15 vibrations per second; but the lower frequencies are difficult to obtain, while the highest value requires relatively fine wire or large tension. The sixty cycle frequency is obtainable with ease, and resonance is sharp. After the frequency has been measured with the impulse counter, it may be checked by the formula for the frequency of a vibrating string.

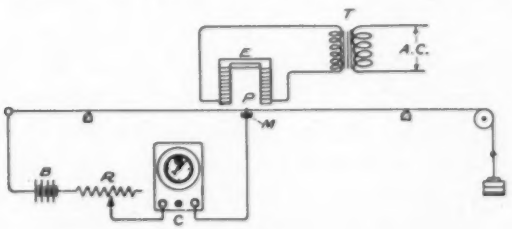

FIG. 4.

A modification of the experiment just described is made by employing the circuit shown in Fig. 5. Instead of using alternating current, as in Fig. 4, the wire is kept in vibration by means of the battery and interrupter PM, which in this case are connected in series with the electromagnet E and the impulse counter C. The frequency, which in this experiment can be varied within wide limits, is fixed by the length and the linear density of the wire, and by the tension. These values

can be measured and substituted in the usual formula, and the computed frequency compared with that measured by means of the impulse counter.

A very practical way of utilizing the impulse counter is to arrange it as an electric stopwatch of high speed, for measuring short time intervals with much greater accuracy than is possible with an ordinary stopwatch. To make this adaptation, the impulse counter is connected to the spark timer, as described elsewhere.[3] When so connected the rate at which the large hand moves over the dial is solely determined by the frequency of the vibrating bar of the spark timer. This frequency may be adjusted to any desired value up to 27 vps. If adjusted, for example, to 20 vibrations per second, the impulse counter reads directly in .05 second divisions, and the large hand makes one complete tour of the dial in 3 seconds. The starting and stopping button on the instrument may be used in this case for timing the interval in question;

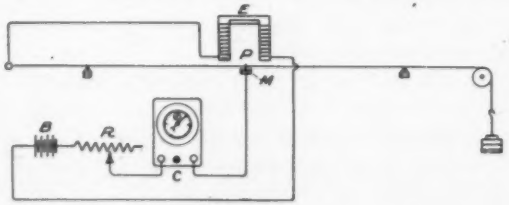

FIG. 5.

but it may be more convenient to operate the counter off the extra contacts of the spark timer, and to close and open the circuit, respectively, by means of a hand-operated key, for starting and stopping the counter mechanism. In place of the hand-operated key, an automatic switch or switches may be used, provided the closing and opening of the circuit can thus be brought about precisely at the initial and final instants, respectively, of the interval being measured.

The two experiments described, in which an iron wire is caused to vibrate by means of an electromagnet, were developed on the basis of Exp. 83 in Minor's "Physical Measurements" and Exp. 119 in Zeleny and Erikson's "Manual of Physical Measurements." To these authors the writer expresses his obligation. He also expresses his appreciation to Mr. M. C. Magarian with whose able assistance the apparatus was developed.

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CHICAGO, ILLINOIS.

[3] Loc. cit.

Television. By H. Horton Sheldon, Professor of Physics, New York University, and Edgar Norman Grisewood, Instrustor of Physics, New York University. x+190 pp. D. Van Nostrand Co., Inc., New York. $2.75.

The remarkable development which the past few years have witnessed in radio seem to justify the assumption that Science's latest prodigy, Television, may soon follow in the footsteps of its older brother. With Television occupying the attention of both the scientists and the layman, this book is indeed opportune. The authors, in attempting to write a book which is both comprehensible to the layman and at the same time sufficiently technical for the scientist and engineer, have undertaken a difficult task. They are to be congratulated for the excellent manner in which they have accomplished this end. Those who are unfamiliar with the scientific principles which underly the apparatus used in wire picture transmission and television will find this book of great value; the scientist or engineer, on the other hand, will find in the second half an excellent summary and discussion of the varjous systems which have been developed. The book has been made authoritative by the cooperation of the individuals and concerns who have produced these systems.

Commencing with a brief, yet comprehensive historical sketch, the authors proceed with short chapters on Optical Systems and the Eye, Electromagnetic Waves, Selenium and Photoelectric Cells, Glow Lamps, Oscillographs, Scanning, and Synchronization. A chapter on Telephotography gives a description of the several systems now in use. The scientist will undoubtedly be chiefly interested in the latter and in the detailed descriptions of the Baird, Bell, Jenkins, and Alexanderson systems of television in the second half of the book. The book concludes with short chapters on Relays, Amateur Equipment, and a prophetic discussion of the future of the field of television.

The authors realize that they are handicapped by the rapidity with which this field is developing and hope by frequent revision to keep the book up to date. In revising, the authors will help the layman if they will make sure that no technical terms are used without previous adequate definition. The book is profusely illustrated throughout with pictures and diagrams. A full page colored plate of the emission spectra of fourteen elements is of questionable value, since the scientist is already familiar with it, and it is not sufficiently explained to have much meaning to the layman.

H. J. REICH